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JET PROPULSION ENGINES AND PROPELLER DRIVE GAS TURBINES—THEIR APPLICATION IN FUTURE AVIATION.*

BY

R. P. KROON,

Manager of Engineering, Westinghouse Aviation Gas Turbine Division.

SUMMARY.

It is the purpose of this paper to give a brief description of the operating principles and major design features of aircraft gas turbines, and to evaluate the performance characteristics of jet engines and gas turbine propeller drives to determine how these new power plants may be applied in future aviation.

OPERATION OF JET ENGINE.

Contrary to the rocket which carries along its own oxygen to burn with the fuel, jet propulsion engines bring in the atmospheric air from the outside (Fig. 1).

Contrary to the "buzz" bomb, the action of the jet engine is continuous rather than intermittent. The principle of the jet engine (or "turbo jet" as it is sometimes called) is very simple and not unlike that of the hot air engine of former decades. It will be recalled that in the hot air engine air is compressed by a piston. The air is expanded by heating it and the expanded air then does work on another piston. Because the specific volume of the air has been increased in the heating process, it can supply more power than what it took to be compressed. This surplus is used to provide useful shaft horsepower.

While in the jet engine the main elements are rotating, and not reciprocating parts, the principle is just the same. Air is pumped in by a compressor. Entering the combustion chamber, the air is heated by buring liquid fuel in it. The expanded hot combustion products are made to drive a turbine, the sole purpose of which is to supply power to

* Presented at a joint meeting of the Franklin Institute and the Institute of Aeronautical Sciences on November 7, 1945.

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keep the compressor and the small accessories going. In the jet engine the useful horsepower appears not in the form of shaft horsepower, but in the form of a high velocity jet, the reaction of which serves to propel the airplane.

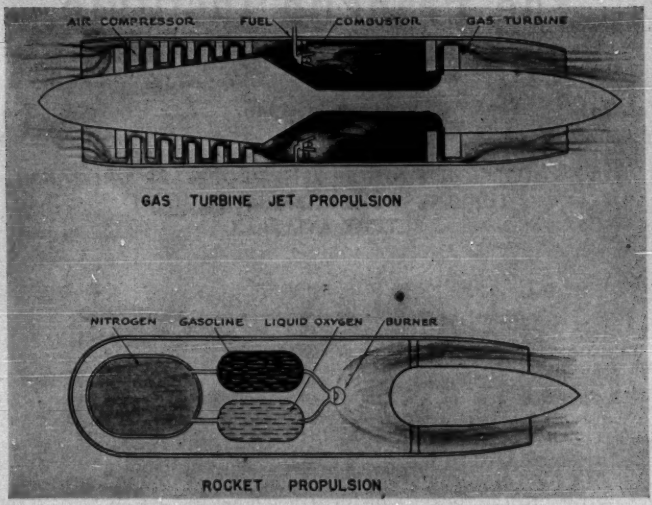

FIG. 1. Schematic diagram of rocket and jet engine.

GAS TURBINE PROPELLER DRIVE.

The term "aircraft gas turbine" is applied not only to the "jet" engine, but also to a propeller drive in which the gas turbine replaces the reciprocating engine (Fig. 2). In this form we have again a compressor, a combustion chamber, and a turbine followed by an exhaust nozzle. However, in this form of gas turbine the turbine element takes out more power than is needed to drive the compressor. This surplus power is used to drive the propeller through suitable reduction gearing. A small part of the energy (about 20 per cent.) remains in the jet, where it furnishes useful power.

DESIGN PRINCIPLES.

Jet engines have been built in several forms. One distinguishes centrifugal and axial flow compressors depending on whether the air particles leave the rotating member in a radial or in an axial direction. While the centrifugal compressor contains one or more vaned impellers, the axial compressor rotor contains a large number of blades, which like

propeller blades, push the air towards the combustion chamber. Either type has found use in jet engines.

Fig. 3 shows schematically the "Whittle" type engine, as built in 1940 on the basis of the suggestions of Group Captain Whittle, who had made concrete proposals for jet engines in England since 1930. The centrifugal design is the one followed by the best known English builders, among whom Rolls Royce and DeHavilland are outstanding.

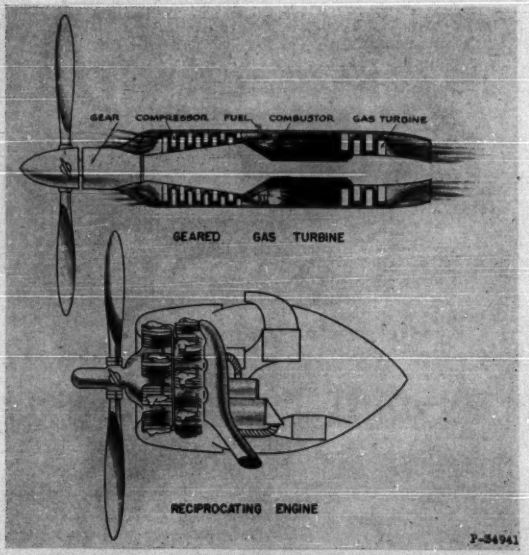

FIG. 2. Piston engine and gas turbine propeller drive.

In this country, the axial flow type of jet engine (Fig. 3) was pioneered by Westinghouse under sponsorship of the U. S. Navy, Bureau of Aeronautics. Independently, the Germans used axial flow compressor on all their jet engine designs known to us.

Jet propulsion makes possible the use of exceptionally "clean" airplanes capable of high speed. In fact, jet propulsion does not offer advantages in performance unless it is applied to high speed aircraft. The higher the flight speed the more important it is to make the airplane just as streamlined as possible. In terms of the engine, this means that a small frontal area (a small diameter) is extremely desirable. The choice of an axial flow compressor permits a design with a small diameter (the two types shown on Fig. 3 are approximately to scale).

In general, the axial flow compressor seems to have a slight edge in efficiency. On the other hand, there appears no question but that the centrifugal compressor is the less expensive to build, and that for very small sizes its efficiency may prove equal or superior to that of the axial flow compressor.

In the centrifugal type engine the air must be turned around—before and after the compressor—several times before it leaves the ex-

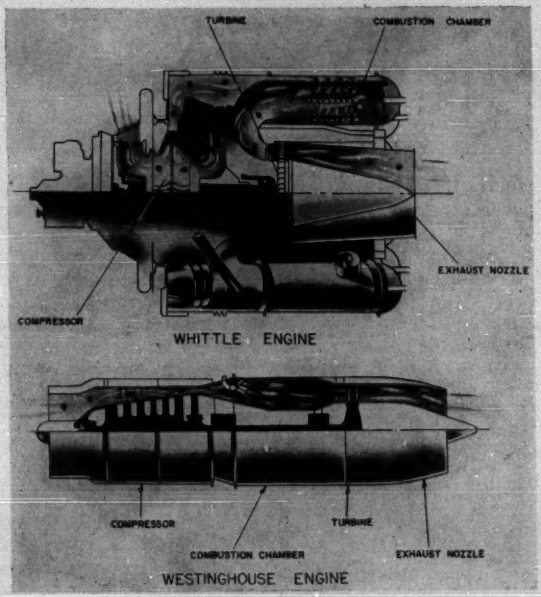

FIG. 3. Whittle centrifugal jet engine (top) and axial flow jet engine (bottom).

haust nozzle. In the original Whittle type the air has to make eight right angle turns before it emerges. Each turning operation involves a loss of energy which is avoided in the "straight through" axial flow design.

An axial inlet to the engine is important because it permits the aircraft designer to bring in the air with a minimum of loss. Thus he can take maximum advantage of the so-called ram effect—the increase in engine inlet pressure resulting from the forward speed of the plane. High inlet pressure results in a high power output and high efficiency of the power plant.

Of interest is also the different treatment of the burner design in the two types of engines shown. In the Whittle type engine combustion takes place in a number of cylindrical burners, called "cans" mounted along the outside of the engine. In the axial flow design, shown in Fig. 3, there is only a single combustion chamber of annular shape.

DESCRIPTION OF ENGINES.

By way of example, several axial flow engines are shown in this article.

The first American designed jet engine, now known as the Westinghouse 19A engine (Fig. 4), was a good example of a streamlined design of small frontal area. This engine, which is good from some 1,300 hp.

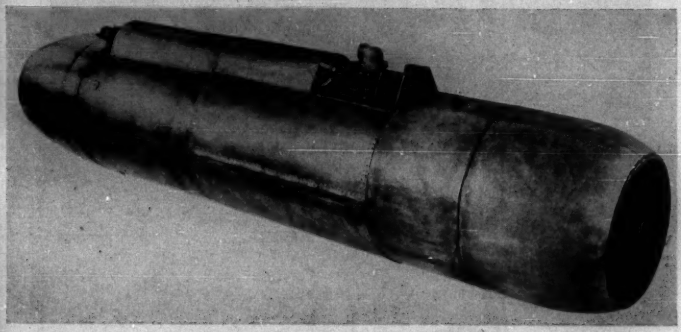

FIG. 4. 19A axial flow jet propulsion engine.

at 450 miles per hour, has a diameter of only 19 in. It was designed to be flown as an exterior booster engine underneath the Navy "Corsair" fighter.

This engine, which is now obsolete, drives only the accessories necessary for its own operation; there is no provision to drive aircraft accessories such as generators, hydraulic pumps or vacuum pumps.

Fig. 5 shows the first 19A engine (without its fairing) and the devoted crew of development engineers and laboratory mechanics responsible for its design and assembly. The engine ran first on March 18, 1943.

Fig. 6 shows the 19A engine as it was mounted underneath the "Corsair." It is of interest to note that the ratio of the diameters of the 1,300 hp. jet engine and the 2,000 hp. reciprocating engine driving the propeller is about 1 to 3.

The 19A engine was the first of a family of axial flow jet engines.

FIG. 5. Engineers, mechanics and 19A engine.

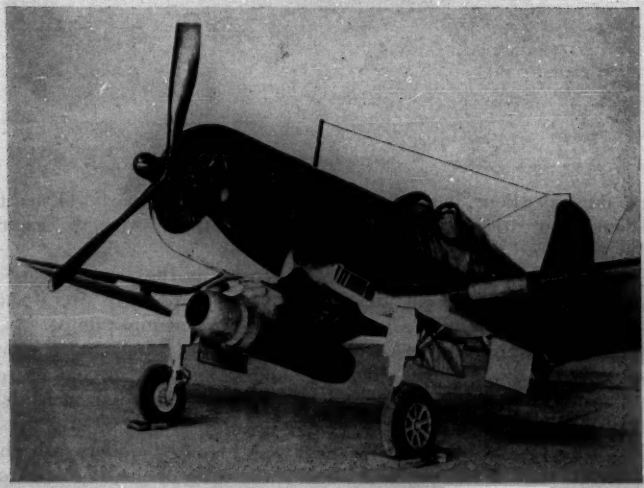

FIG. 6. Navy "Corsair" fighter carrying the 19A engine underneath the fuselage for flight testing. The first flight was made in January 1944.

Fig. 7 shows its successor, the Westinghouse 19B ("Yankee") engine, rated to give a test stand thrust of 1,365 lbs. This amounts to some 1,400 hp. at 450 m.p.h. and 1,700 hp. at 550 m.p.h., sea level. The basic diameter of the engine is 19 in., the weight, complete with all accessories for aircraft use, slightly over 800 lbs. The rotating elements—the compressor and the turbine—whirl at a maximum speed of 18,000 revolutions per minute. At this speed the blades travel at rifle bullet velocities of a round 1,200 ft./sec. At such velocities the centrifugal force on each blade amounts to 50,000 times the weight of the blade.

The engine swallows air at the rate of 30 lbs. per second, corresponding to something like a million cubic feet per hour.

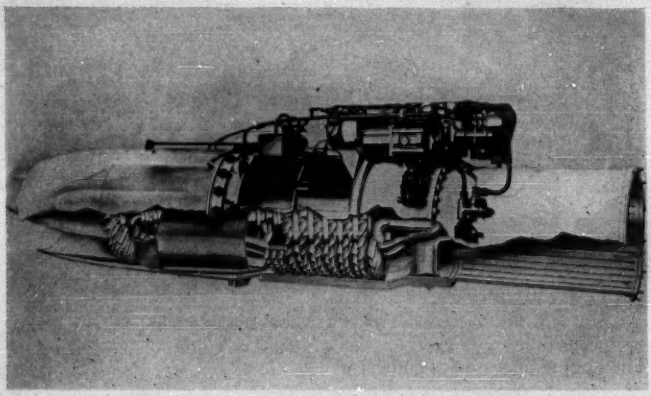

FIG. 7. Cut open view of 19B axial flow jet engine.

As the air enters the engine, it first performs an important little chore in cooling the oil, used for the lubrication of the three main bearings of the engine. The aluminum oil cooler is located in front of the engine where it is subject to cooling air whenever the engine runs, independent of whether the airplane is flying or on the ground.

The 3,400 hp., six stage axial flow compressor pumps up the air to about three times atmospheric pressure in about .004 seconds. The compressed air then enters through the perforations of the annular combustion chamber. Fuel is brought in through a rows of atomizing spray nozzles. Spark ignition is used, but as soon as the fuel is burning the ignition system can be turned off because combustion is then continuous.

The air particles spend only .01 second in the combustion chamber.

In this short time the energy released per cubic foot by the fuel is about 1,000 times as much as in the conventional power plant boiler.

The combustion products enter the turbine at temperatures up to 1500° F. In the turbine they give off about two-thirds of their useful energy to drive the compressor. The jet emerges from the exhaust nozzle of the engine with a velocity of around 1,200 miles per hour.

The 19B engine carries accessories to serve the aircraft as well as accessories necessary for the operation of the engine. The accessories are mounted on a gear case where they are easily accessible. They can be located on top, on either side, and at the bottom of the engine, wherever they can be faired in best within the contour of the airplane.

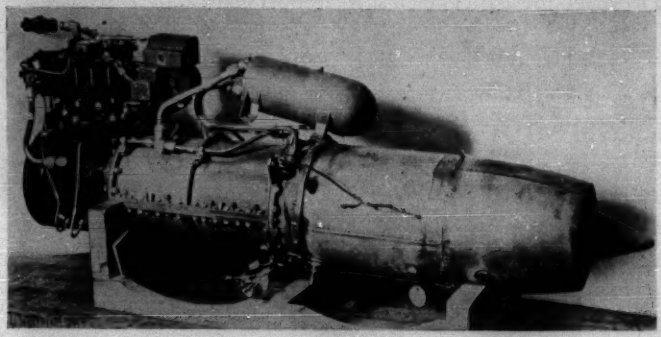

FIG. 8. Outside view of 9.5A axial flow jet engine.

The engine accessories consist of—

(1) An electric starter to bring the engine up to the speed at which it can maintain operation.
(2) A fuel pump to deliver fuel to the combustion chamber.
(3) An oil pump to circulate the lubricating oil.
(4) An overspeed control to prevent the engine from "Running away."
(5) An electric tachometer to give a visual indication of r.p.m. to the pilot.

The aircraft accessories are—

(1) A generator to provide electric current.
(2) A hydraulic pump to furnish high pressure oil to operate landing gear, wing flaps, etc., or a vacuum pump to operate the aircraft instruments.

The Westinghouse 9.5A engine, shown in Fig. 8, is in many ways a small model of the 19B engine. As its name indicates, its diameter is

only 9.5 inches. Its static thrust is 275 lb. and its weight is 145 lbs. The top speed is 34,000 revolutions per minute. The engine has anti-friction bearings lubricated by an oil mist (fine oil particles carried by compressed air). The 9.5A engine is equipped with an all-speed control.

The 19B engine came to test first in April 1944. The 9.5A engine followed in June 1944. Further improvements are made almost day by day with the result that, at this time, engines of this same type are counted the lightest and most efficient in the United States and, for the same power, the smallest in the world.

EVALUATION OF ENGINES.

In general aircraft gas turbines are evaluated in the same way as up and down engines, fuel consumption, weight, and size being the most important characteristics.

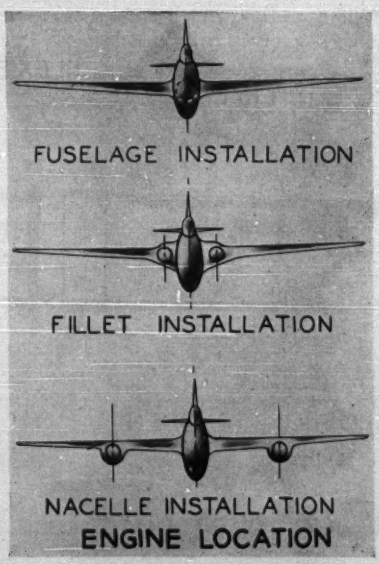

FIG. 9. Three types of jet engine installations.

It is interesting to compare the German engines, which have received a great deal of publicity, with the engines produced in this country. The Germans have done a creditable job of getting engines into the air with limited means, particularly with a very limited choice of high temperature alloys. In their designs they have applied a few ideas that we can well afford to take over. But, largely because of

material limitations, the Germans had to go to complicated means of cooling and low stresses resulting in large engines. Altogether the weights of the German engines run 1½ to 2 times as high as those of American engines of about the same power. Because of the large amount of compressed air used for cooling, some 15–20 per cent more fuel was consumed. On the basis of the same power the diameter of the German engines is 25 per cent larger than that of the engines which are shown in this article.

COORDINATION OF AIRCRAFT AND ENGINE.

Jet engines can be mounted within the fuselage, in the fillet of the wings or further out along the wing (Fig. 9). Contrary to the piston engine-propeller combination, the jet engine usually passes air all the way through the vital construction members of the airplane, such as the fuselage or the wing.

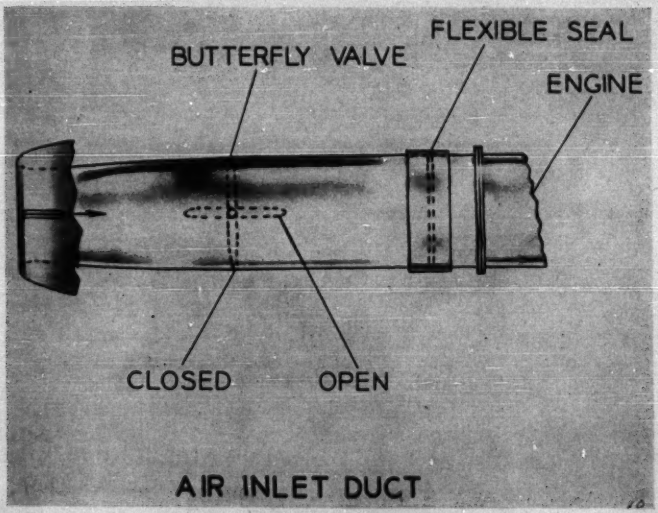

FIG. 10. Typical aircraft gas turbine inlet duct.

In any jet application close cooperation of aircraft builder and the engine designer is, therefore, of the greatest importance. After all, it is the combination that is either a success or a failure.

The question of overall performance is, of course, a vital one. Any jet engine, whether it is axial or centrifugal, is sensitive to inlet and exhaust conditions. A 10 per cent loss in total pressure at the engine

inlet results in a 20 per cent loss in thrust. Wherever ducting is needed, such as in a fuselage installation, it must be carefully designed so that it will not ruin engine performance and thereby the overall performance of the aircraft.

By way of example, let us take a high speed fighter with a short inlet duct to the engine (Fig. 10). For an airplane of this sort the builder will want to use a small inlet area for the duct. The engine designer on the other hand may inadvertently be thinking in terms of a fairly large diameter for his compressor inlet to keep his internal losses low. Unless the two get together at an early stage, the aircraft builder may be forced into trying to make a duct that diverges too rapidly, resulting in low duct efficiency and poor performance.

FLIGHT PERFORMANCE CHARACTERISTICS.

The jet engine serves essentially to produce a stream of hot gases with high velocities. It has been called a "hot gas generator."

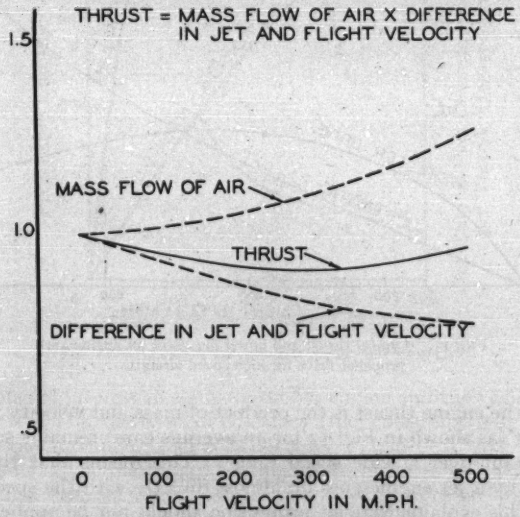

FIG. 11. Variation of jet engine thrust with speed.

The forward thrust exerted by the engine is a direct result of the change in momentum of the air mass absorbed by the engine. Forces are needed to accelerate the air particles through the engine. It is the reaction to these forces which we call thrust.

To investigate how the thrust of the engine varies as the flight speed increases, we must know what happens to the mass of air transported through the engine and what happens to the inlet and exit velocities of the medium.

As Fig. 11 indicates, with increasing flight speed the air in front of the engine becomes more and more compressed due to the "ram" effect. Therefore, at the same r.p.m. and with denser air, the transported mass *increases* as the plane goes faster.

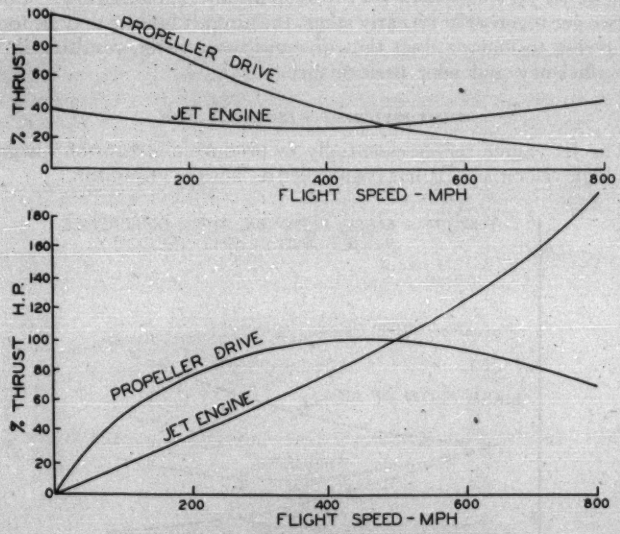

FIG. 12. Typical thrust and power curves of jet engine and propeller drive for high speed aircraft.

Since the engine thrust is the product of mass and velocity increase, the thrust (as shown in Fig. 12 for an average case) remains substantially constant over a wide speed range. This means that the power delivered by a jet engine goes up almost directly with the speed of the plane. This explains that jet propulsion should not be applied to low speed aircraft.

If we now compare this performance with that of a variable pitch propeller—be it driven by a reciprocating engine or by a gas turbine—we find that such a drive will give substantially constant power over a wide range of speed. At take off such a propeller drive will give some three times the thrust realized at maximum speed.

These facts disprove the popular misconception that jet planes have a phenomenal take off—quite the opposite is true. On the basis of the same power at high speed, the take off thrust of the jet engine is considerably less than that of a propeller drive and a longer take off distance is needed.

What has confused most people is that the jet engines that can be made available for aircraft such as small fighters are so high powered that their take off is still acceptable. However, if the same horsepower could be put into a propeller drive the take off distance would be even much less.

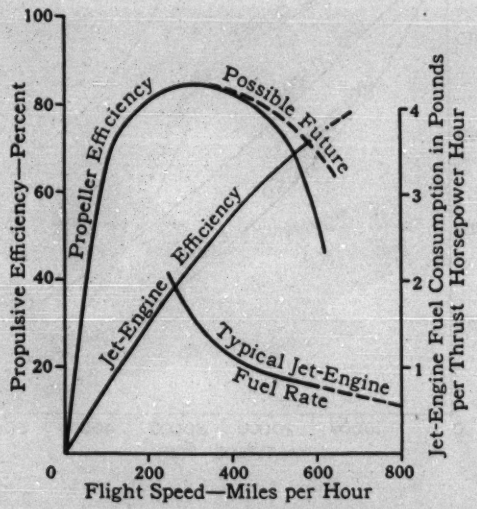

FIG. 13. Efficiency curves for propeller and jet engine.

It is also of interest to compare the jet engine and the variable pitch propeller drive on the basis of efficiency (fuel consumption).

Since the amount of fuel does not vary a great deal with flight velocity it is mostly a question of heating up the air mass to a safe limiting temperature—the jet engine efficiency will vary more or less with the power output, and as we have seen for speeds up to 800 m.p.h. the power output varies almost directly with the speed of the plane. Therefore, the efficiency of the jet engine increases as the plane goes faster (Fig. 13). On the other hand, with a variable pitch propeller and for a given power, the efficiency of the drive remains substantially constant over a wide speed range until propeller tip velocities are reached that are in the range of the velocity of sound. Then the efficiency drops

abruptly. There is a point, at perhaps 550 m.p.h., where the jet engine efficiency exceeds that of the propeller drive. It is beyond this point that the jet engine really comes into its element.

The altitude performance characteristics of the gas turbine, compared with those of the reciprocating engine (with and without supercharging) are indicated in Fig. 14.

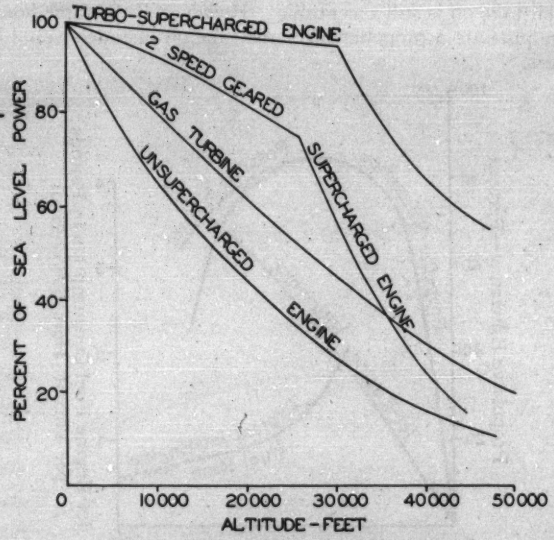

FIG. 14. Typical thrust variation with altitude.

The aircraft gas turbine as such cannot be supercharged. Hence the variation of engine output with altitude is very similar to that of the unsupercharged piston engine. That the gas turbine output does not drop off as rapidly as that of the reciprocating engine without supercharger is largely due to the fact, that the low air inlet temperatures at altitude have a greater effect in the case of the gas turbine cycle.

Modern jet engines have only about one-third the installed weight of a reciprocating engine. This weight saving can be used to carry more fuel or to increase the payload. Similarly, in connection with Fig. 14, and considering a propeller drive on the basis of the same engine weight a gas turbine could be selected to have the same power as a supercharged piston engine at say, 40,000 ft. This gas turbine would then have about twice the power of the piston engine at sea level.

There are other tangible advantages in favor of the jet engine and gas turbine. These are low oil consumption, very small amounts of cooling air required, negligible vibration.

There is the fact that safer fuels can be used. Fuels such as household fuel oil are well suited for gas turbine use.

THE PLACE OF GAS TURBINES IN MILITARY AND COMMERCIAL AIRCRAFT.

We can now attempt to predict where jet engines and gas turbine propeller drives are most likely to be used, based on their typical characteristics. It should be remembered that at low flight speeds the

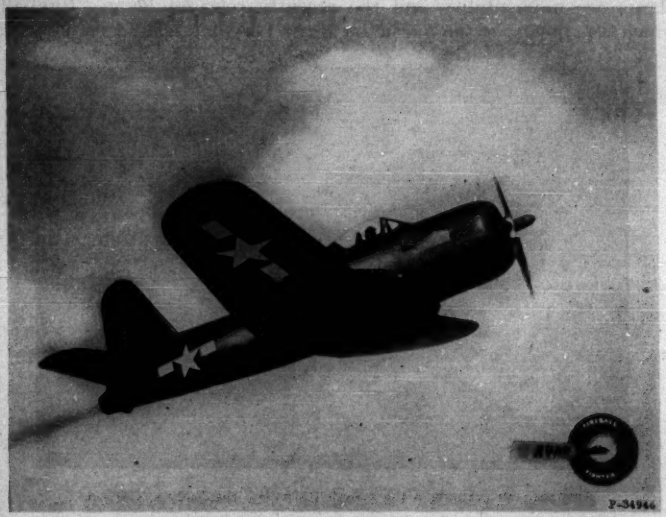

FIG. 15. Ryan "Fireball" Navy fighter, example of composite power plant (piston engine-propeller drive and jet engine).

efficiency of the jet is poor. We must also remember that to propel the plane at high speeds requires high fuel consumption because of the larger force required to pull the plane through the air at such high speeds.

Since the jet engine recognizes no speed barrier—its performance improves with flight speed even beyond the velocity of sound—its use is particularly indicated for high speed fighters and interceptors.* The

* It should be pointed out that there is a limit, well above the velocity of sound, at which the temperature increase due to the ram effect is so great that the additional increase in the compressor brings the air temperature up to the maximum allowable peak temperature for the engine. Thus it would not be possible to burn any fuel without exceeding this temperature limit.

jet plane has also decided possibilities for short, fast transport. Besides having the advantage of high speed—and after all that is the main advantage that aviation can offer—such jet driven passenger planes, due to the almost total absence of vibration, and due to the fact that very little noise is transmitted to the plane itself, should prove exceedingly comfortable. In such applications the high fuel consumption of the jet engine would be offset by light engine weight, low initial cost, and higher cruising speed, making possible a larger number of trips per plane in a given time.

Combinations of the reciprocating engine and jet engine power plants are a possibility and the recently released Ryan "Fireball," which was flown for the public on Navy Day has such a piston engine

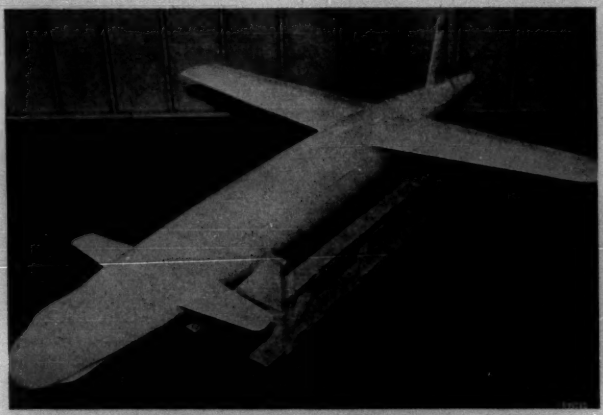

FIG. 16. An example of the "Guided Missile," the Navy's "Gorgon."

propeller drive and jet engine combination (Fig. 15). With such a combination cruising only on the reciprocating engine is possible with good fuel economy. The power of the jet engine can be added for bursts of power during take off and in a fight.

The jet engine offers also considerable promise for the so-called guided missiles whether they take the form of pilotless planes or projectiles without wings. While being a more complicated mechanism than the buzz-bomb, the efficiency of the jet engines is at present so much higher that they have an advantage for missions where range is of the utmost importance. A typical example of these guided missile developments is the Navy's "Gorgon" recently released (Fig. 16).

It should be realized that with the present day large piston engines at the high speeds fully 30 per cent. of the engine power is lost in drag-

ging the engines with their nacelles through the air and providing cooling for the engine. This has been called the "barn door" effect. In contrast, with the much smaller gas turbine (Fig. 17) this figure is reduced to 10 per cent. Our analysis of gas turbine propeller drives has proven that gas turbines can be built with only one-half the diameter of the reciprocating engine for the same power. We are confident that gas turbines can be built to have an installed weight between one-half and three quarters of the installed weight of the piston engine.

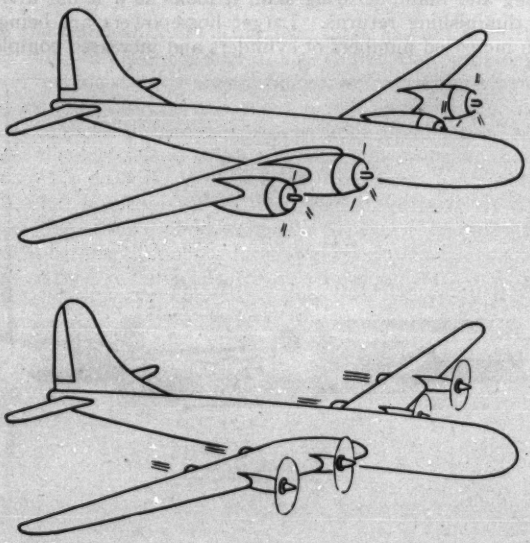

FIG. 17. Large aircraft driven by piston engines (top) and gas turbines (bottom).

Already with present day compressor and turbine efficiency, the gas turbine has a better fuel consumption at full load than the large reciprocating engine. On the test stand its cruising part load efficiency is not as good as that of the reciprocating engine. However, taking into account the advantage of its smaller diameter (meaning less drag) and its lighter weight, we find that for a large airplane the gas turbine would be superior in every respect—including range or pay load, speed, rate of climb, and take off. For any conditions except extremely high speed and low power the gas turbine propeller drive appears as the power plant of the future.

Fig. 18, while marked "Westinghouse Propeller Drive Gas Turbine," should not be taken too literally. It is more in the nature of an artist's

conception. It indicates what an efficient streamlined aircraft gas turbine would be expected to be like.

LARGE ENGINES—FUTURE ACTIVITY.

Throughout aviation history and particularly in the military field, the trend has been towards faster airplanes and consequently towards more powerful engines. We have now arrived at the cross roads.

On one side is the up and down engine. While being a marvel of engineering and manufacturing skill, it looks as if it has arrived at a point of diminishing returns. Larger horsepowers are being bought only with increased numbers of cylinders and increased complexity.

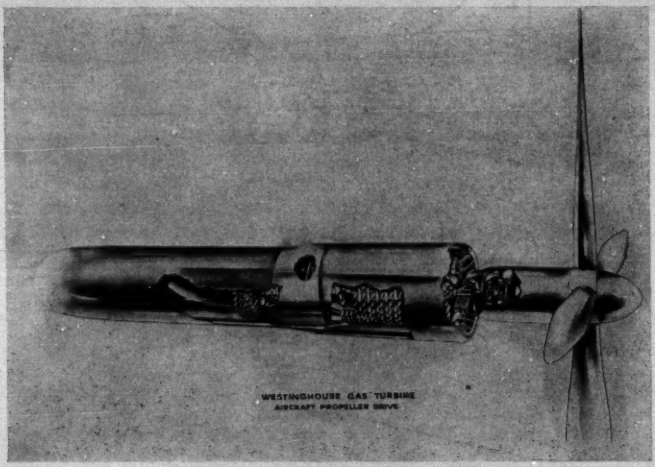

FIG. 18. Artist's conception of gas turbine propeller drive.

On the other side is the aircraft gas turbine. It does not seem to recognize any size limits as yet, except for the practical limits of getting certain sizes of forgings and castings made. Larger gas turbines will not necessarily require any more parts and they will become more efficient as the size increases.

Further increase in flight speed will favor the jet engine. Further increase in power will favor the aircraft gas turbine in jet or propeller drive form.

Already today there is a demand for engines of 6,000 hp. and 10,000 hp. The Germans built an 8,000 hp. jet engine. A piston engine for such powers would be a very difficult piece of machinery to build, and it is doubtful if its size and complexity would not be prohibitive. But

we see no reason at all why jet engines or gas turbines of 6,000 or 10,000 hp. cannot be built immediately.

There is no question but that gas turbine development is still in its infancy. So far, only the simplest form of turbo-jet has been worked out in a practical manner. There is no question but that, within a very few years, there will be refinements added—intercoolers, reheaters, heat exchangers. The properties of the jet engine will be utilized for high speed maneuvering and to perform chores like cabin heating and de-icing.

Within the framework of this article, there has not been the space to include metallurgical considerations, combustion problems, aerodynamics of turbine and compressor blades, high speed bearing developments and many other subjects, each of which is fascinating for its own sake. Yet the picture is not complete without a realization, how enormous a field of research is to be covered, and how intensive the research must be to make good the promise that the aircraft gas turbine holds out. This need is recognized by the Armed Services. It calls for a good deal of effort from organizations like the N.A.C.A. and the Bureau of Standards, besides that of the industrial concerns which are devoting a large part of their research effort to this development. It is only with such intensive and coordinated research that a leading position in this field can be retained.

Trace "Tagged" Atoms.—"Tagging" atoms by using the cyclotron machine to make the atoms radioactive is one of the newer tools of plant research. By tracing an irradiated or "tagged" chemical as it moves within the plant, scientists of the U. S. Department of Agriculture and the Ohio Agricultural Experiment Station think they will be able to explain some fundamental questions not answered yet. They hope for explanation of such puzzles as the formation and action of chlorophyll—the green coloring matter of plants—and how starch is formed from the carbon dioxide the plant draws from the air and water from the soil, and how and where starch changes to sugar.

With the cyclotron machine, Dr. J. D. Sayre of the Agricultural Research Administration explains, it is possible to make many chemicals temporarily radioactive in much the same way that radium is radioactive. A delicate laboratory tool, the Geiger Counter, is so sensitive to radiations that it registers the presence of a single charged atom. This combination permits the tracing of "tagged" atoms as they move within a plant. Fortunately, even after irradiation, atoms still combine chemically in the same way that normal atoms do.

Thus, if a scientist uses an irradiated chemical—phosphorus, for example—in a solution around the roots of a plant, he can use the "counter" to measure the number of irradiated phosphorus atoms passing up the stem of the plant. By extending such studies of "tagged" atoms it should be possible to find out where in the stem—or leaves or fruit—of a plant the phosphorus ceases to move. This would point out an area worth study to discover whether the phosphorus had entered into a new chemical combination—and the nature of the new combination. A man wise in plant chemistry might then be able to answer some baffling questions.

Research of this type calls for almost unlimited skill, precision and patience—often for teaming of several specialists; for knowledge and imagination in interpreting results. Full answers to the big questions may lie far in the future. Meantime, experience with other basic research shows that partial answers may solve practical problems and make such work highly profitable. The Research Administration points out that scientists know they are still only on the threshold of studies of growth regulating substances or hormones, but the practical gains from hormone sprays (used to prevent the drop of apples before they are ripe) far exceed the cost of all the hormone research; and already there are several other by-products in the rooting of nursery cuttings, in weed eradication, in production of unfertilized and seedless fruit, and in other fields of plant development.

R. H. O.

A NEGLECTED ASPECT OF RESEARCH.

BY

THOMAS COULSON.*

During the last few years of peace-time industrial activity, the annual reports of the Secretary of Commerce revealed that each year about 15,000 patent applications were abandoned and allowed to perish.

Behind this somewhat obscure statement lies a story of economic waste that calls for a remedy. At first glance, it would appear that because the loss of a half million dollars in fees, paid to the Patent Office, is spread over a large number of individuals and companies, no one need be gravely concerned in finding a remedy for the situation. However, a moment's reflection ought to be sufficient to prove that the modest amount lost in fees is an insignificant fraction of the loss incurred in laboratories where the research was conducted upon the abandoned applications. This loss must run into millions of dollars.

The average business executive and research worker might do well to ponder upon his company's current procedure and to inquire what is its contribution to this not inconsiderable amount of waste, actual and potential. It is notorious that some business executives who claim to support the principle of research and development are nevertheless prone to restrict their research programs within the narrow limits of product improvement. This type of executive readily supports an appropriation for an investigation that will lead to an improvement in the setting of a bearing or of a gear train, but he raises opposition to the suggestion that the company should spend a large sum of money upon a program designed to develop something fundamentally new. This attitude is commonly attributed to a failure to appreciate the true functions of research. It may, however, arise from an unsatisfactory encounter with the Patent Office examiners after a long and expensive laboratory research program. It is unfair to accuse these cautious men of being actuated by motives that betray a lack of appreciation for research.

Unconsciously these men realize there may be an imperceptible defect in its conduct. In their eyes, research is not necessarily a gainful enterprise, as it should be, and is when properly conducted. Until they perceive the nature of the pit-falls they will not give research unqualified support.

Another unfortunate attitude is adopted by some executives and their disgruntled research workers. They denounce the patent system

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as the author of their disappointments. They declare that it handicaps research and should be revised in favor of the inventor.

Very obviously, all patent applications are not abandoned for a solitary cause, but the commonest cause for rejection is because the application wants originality. The claims either infringe upon earlier patents or the material on which the claims are based has already been described in the technical literature. When such rejection occurs the fault lies too often with the research worker, who has regarded the laboratory as the sole agency for applied research activity.

Let us see what happens in so many cases when a research program is initiated. The man to whom the task is assigned may be presumed to have a general knowledge of the subject. He requires to supplement this with more specific information. His first step on being assigned the research problem is to make a survey of the literature. It cannot be too heavily stressed that the more thoroughly he makes this survey the less likely is he to become involved in duplicated experimentation and development.

Two hazards commonly attend this preliminary work, upon which so much depends.

The first is represented by the mental attitude of the worker toward his task. To a great many research workers the laboratory is the sole field of operations. These men live in their test-tubes; they are the super-laboratory technicians. The sole purpose of this type of worker is to glance hurriedly through some of the available literature in order to uncover a few leads which will assist him in the attack upon his individual problem. With a prayer, a hope, and a few notes, he hurries back to his laboratory to prepare for the performance of his task.

The second hazard arises from an insufficiency of information. The more well-intentioned worker, resolved upon making his survey as complete as possible, not restricting himself to the investigation of his direct needs, but prepared to penetrate into the bypaths that communicate with allied areas of research, finds himself frustrated by not having access to a well equipped library. If the determination to avoid the pit-falls of the prior art is to be successfully followed, workers must be furnished with adequate facilities for library research.

It should be remembered that research work is now more active than it has ever been, and publication of results is more generally practiced than ever before. Any scientific subject, either basic or applied, is speedily segregated into its component parts for investigation, and a missing link in the long meandering chain of research might easily lie concealed in some obscure corner off the beaten paths of investigation.

But, without further digression, let us see how these two types of men progress in their applied research. Beginning with a few notes they embark upon their laboratory program. As this progresses, they accumulate a volume of observations from which, in the fullness of time,

it becomes possible to abstract the material that will form the substance for claims to a patent. This is turned over to a patent lawyer who prepares the application for the patent and, as some cynics observe, to dress it in the incomprehensible language of the patent law.

Approximately one in three of these applications die at this stage. They perish as soon as the examiners decide that there is no ground upon which a patent may be awarded, because the material disclosed in the application is either covered by an earlier patent or it has been described in the technical literature. If the patent is forthwith abandoned it represents so much sheer waste of effort and money. The sum total of this waste is incalculable, but it must amount to several millions of dollars annually.

But that is by no means the end of the story. Laboratory research performed without adequate preparation does not necessarily result in an abandoned patent application. Although the basic claims of the application may have proved to be unsound, yet the course of the investigation may have brought to light some entirely new features which require protection. The patent lawyer re-enters the picture and undertakes to whittle down the claims until an acceptable compromise is reached with the Patent Office examiners. This process incurs additional expense, for it may be prolonged over several months. Rare and happy is the research worker who escapes the chagrin of having his patent claims whittled down at some time or other. Others come to regard the process as the inevitable outcome of all their research. Indeed, these tugs-of-war between lawyers and examiners are so characteristic of the patent system that cynics have concluded the system was devised, not to reward inventors, but to enrich lawyers. They give point to the allegation that research is only "educated gambling."

No attempt has been made, to my knowledge, to arrive at an estimate of the expense involved in the whittling process, but when the costs of the duplicated labor are added to the extra legal fees, the sum total must be an impressive figure. When this is added to the waste represented by the abandoned patent applications we begin to perceive the gravity of the consequences involved in an incomple preparation of the initial stages of the research program.

The most tragic feature of the examinations which lead to the rejection by the Patent Office of so many submitted claims is, that the process of the examination bears within itself a suggested remedy for all the implied waste of time, effort, and money. The examiner's knowledge upon which rejection is based is derived solely from a review of the literature on the subject. These men have no exclusive sources of information; they rely entirely upon publications available to any members of the public. The only distinctions between their library research and that of the applicant for a patent reside in the fact that the examiners are able to evade the two inherent hazards in this phase of

the research. They have adequate facilities at their command, and they devote an appropriate amount of time to the examination of the literature on the subject.

Here, then, are the means of checking a major portion of the waste in fruitless laboratory research. Granted a well equipped library and the will to make a painstaking survey of its contents before practical work is opened, and little imagination is required to perceive how the number of abandoned and amended patent applications can be greatly reduced. The more initiative and resourcefulness displayed in the initial stages of the work, the more profiatble will be the laboratory work.

Of course, the harassed business executive who keeps a watchful, and often a jaundiced, eye upon the costs of research will be prone to protest against employing a highly paid specialist in experimental work to perform library research work, for which he may have neither taste nor talent. There is a certain amount of validity about this argument, but it cannot excuse the omission of thorough library research. If a man is not available, he must be found, otherwise the company will be confronted constantly with the specter of waste that lurks within all research liable to duplication.

Let us not deny the element of truth in the assertion that the practical worker is not always best qualified to conduct the preliminary library research. Much depends upon his mental approach.

If the research worker approaches his task with a predisposition to accumulate technical data on which to base his own projected investigation, the chances are this will subconsciously dominate his reading and influence his interpretation of the accounts given by earlier workers in the field. He will fail to read between the lines. Having found some shred of information that will serve to furnish him with a lead, he will fail to pursue the examination that would show him the material had already been put to practical use. It may be protected by a patent on a process or a device, the derivation of which is not openly apparent from the written description of the experimental work.

Then again there is the danger arising from over-anxiety to engage in practical work. This is manifest in the worker's readiness to abandon further research when he has once gained a lead into his own investigation. Unless this type of worker firmly resists the temptation to curtail his survey of the literature, he is inviting duplication of effort. The time to ascertain whether there is duplication is before an investigation starts, not after it is concluded.

In addition, there is always the risk that the company fails to provide for its staff the means for making a thorough survey of the appropriate literature. Provision is nearly always made of a collection of books and periodicals which embraces the literature covering the firm's direct

problems. Too often this collection comprises no more than the minimum number of books and journals required to keep abreast of competitors' activity. No encouragement is offered to pass beyond the selection so that an investigator may survey the broader implications of research. The allied fields of research are neglected because the gleanings are supposedly small. This deliberate limitation of effort constitutes a major defect in practical work, since there is very little check upon duplication of effort. It is most often the casual gleanings of remote fields of research which present the pit-falls of the prior art.

The temptation to recoil from making a thorough survey of the literature is a real one to a practical man. The research student of a hundred years ago experienced no difficulty in covering the scientific and technical literature of his day. Discoveries and inventions were yet so comparatively rare that the student was able, without undue strain, to march abreast of the knowledge of his fellow workers. But the tempo of discovery and invention has so quickened that a flood of literature is needed to follow the rapidly succeeding developments in research. The increase in specialized industries has produced an extensive technical literature. This has expanded during the past fifty years to such an extent that it has outgrown the capabilities of the man with other employment to keep it under constant review.

A significant feature of the period during which this volume of literature has expanded has been the corresponding growth of abandoned patent applications. A study of the two curves which show the number of applications filed and the number of patents granted since 1880, reveals a gradually enlarging No Man's Land between them. From 1910 to 1930 one application was abandoned for almost every patent granted. Research wastes here must have been appalling. Since the year 1930 there has been a marked improvement but the figures for abandoned patent applications are still too high for satisfaction.

Research is expensive. Money is needed to provide the brain-power and the tools for its undertaking. Unless it brings results, unless it makes profits, it must cease. The most recent figures available show that nearly $350,000,000 is spent annually on industrial research. Industry will soon be appealing to the investing public for funds to revive enterprise and to facilitate reconversion and, as the investing public has already learned, research is as essential to invention as invention is essential to industrial progress. Investors have learned to look for comments on research programs in company reports. Indeed, research has so fully justified itself by results that great expectations are cherished it will step into the billion dollars a year category within a short time. But on one will willingly invest in research to produce inventions unless there is an assurance that the knowledge disclosed by research can be protected by a patent. When seriously regarded, that No Man's Land between the curves of patents applied for and patents

granted during the past fifty years is sufficient to arouse a certain disquiet.

No claim is advanced that more careful investigation of the literature will eliminate all duplication of effort, nor will it ensure that all applications for patents will result in awards for the claims advanced, but enough has been said to show that neither the research workers nor the companies who employ them can divest themselves of the responsibility for making a thorough preliminary survey of the literature which is the surest protection against the waste of duplicated effort.

The best way to ensure finality in this work is to employ specialists in library research, men who are not influenced by the urge to begin practical investigation before a complete survey of prior work has been conducted. With an unprejudiced mind freed from the allurement of laboratory work, this type of worker can read between the lines of published disclosures. He can be trusted to make a closer examination of patents already issued.

Not all research undertakings can afford to supplement their staffs with a full-time specialist for this work, but once the importance of this phase of research is recognized it will be found advisable to engage the temporary services of a qualified searcher. The essential thing is to realize that there must be a proper balance between library research and laboratory research if avoidable duplication is to be prevented.

It should be recognized that the problem is not confined to the matter of personnel. The best qualified worker will be ineffective if denied the proper tools. He must have access to a good library. Here we encounter a conflicting tendency in laboratory location. We hear the argument in some quarters that the laboratory ought to be removed from the noisy and dusty atmosphere of the production plant and should be located on a site which ensures quiet and freedom from interruption. While fully sympathizing with the spirit of this change one cannot refrain from recommending that serious thought be given to the remoteness of the site from a well-equipped library. A suburban location will be no serious handicap, but a rural setting ideally designed to promote the contemplative spirit implies the provision of self-contained library facilities.

The small company labors under the greatest handicap in the matter of furnishing adequate library research facilities. It does not and cannot expect to maintain its own library upon a scale commensurate with its special requirements. Yet, the research workers of such companies must have recourse to the same literature as those of the larger companies. The field of literature to be searched is not proportionately reduced to the size of the company.

With this need manifest, it is surprising that the smaller companies do not make more extensive use of the rich resources of those libraries, like that of The Franklin Institute, which have made adequate pro-

vision of technical and scientific literature over a number of years, and so have plentiful files suitable for searching. Almost every one of these fine libraries tell the same story of a lack of appreciation by those very research staffs who could derive most benefit from them. Where a company cannot afford to provide the necessary library facilities for its research staff, it can, at least, make arrangements whereby a member of the staff can be admitted to explore the resources of one of the large special libraries. The resources of all but the very largest and best of the public libraries are inadequate. There must be an ample patent file and abundance of technical literature. If the librarian is sensible enough to make an effort to collect technical pamphlets, the gain is so much greater.

Finally, while it is acknowledged that the laboratory is the place where printed statements become living facts, it is most unwise for executives to protest against research workers reading technical literature during working hours. It is only through the free admission that there must be a proper balance between library research and laboratory research that any company, large or small, can hope to liberate itself from the bogey of duplicating effort which inevitably results in abandoned or amended patent claims.

Protecting the Big Dams.—Keeping silt from filling the great storage reservoirs of the West—and so making them useless—is a job which can be done by upstream engineering and land management in the postwar era, according to Carl B. Brown of the U. S. Department of Agriculture. He points to successful protection of Lake Mead from silt carried from a tributary of the San Juan river which drains a 65,000 acre area on the Navajo reservation. This stream had been sending 600,000 cubic yards of soil a year down river toward the reservoir until Soil Conservation Service workers moved in ten years ago and built a series of low barrier dams and a system of water spreading dikes which also resulted in spreading of the silt in run-off waters. For this area the SCS development was successful in cutting off all the sediment that would otherwise have helped fill Lake Mead. The water spreading also improved forage production.

Soil that cannot be kept on the land by erosion control methods might better be "trapped" at sites of low value, and so kept out of the great storage reservoirs that are, as Brown emphasizes, "irreplaceable." It is important, Brown says, for the public to realize that it is good policy to protect the large investments in these key reservoirs by spending now the smaller sums that will prolong the usefulness of the great reservoirs. Engineers know that these reservoirs are filling much too rapidly and that most of them cannot be expected to remain useful for more than 200 years unless more of the sediment is kept out.

Low barrier dams in tributary streams have proved effective and would be economical in the long range, Brown emphasizes. These barrier dams are not intended for water storage. They are silt traps, built to allow eroded soil to fill the vast arroyos and to settle on land of relatively low value. Land behind the dams that would be buried by the trapped silt would have only a small fraction of the value of storage in the main reservoirs that would be preserved by holding back the sediment.

R. H. O.

LIGHT DISTRIBUTION FROM RECTANGULAR SOURCES.

BY

PARRY MOON AND DOMINA EBERLE SPENCER.*

1. INTRODUCTION.

Rectangular light sources are of considerable importance in illuminating engineering. The lighting of modern rooms is often effected by rectangular panels of diffusing glass built into the walls or ceiling and made luminous by incandescent or fluorescent lamps concealed in boxes behind the panels. The distribution of light from such sources must be calculated in the design of the lighting system. In indirectly lighted rooms, the entire ceiling may be considered as a rectangular source. Light that is reflected from walls can often be handled in the same way. Natural lighting by means of windows or skylights is also ordinarily treated by the method of rectangular sources.

R. A. Herman seems to have been the first to derive a formula for pharosage (lumen $m.^{-2}$) produced by a uniform rectangular source. His work was published [1] in 1900. In 1909 and 1910, Bassett Jones [2] worked on the subject. In 1924 Yamauti [3] developed the equations independently, and in 1925 Higbie [4] did likewise. A masterly presentation of the basic methods was made by Stevenson [5] in 1932. Higbie and his collaborators have done much in the application to practical problems and in the development of simplified methods of calculation.[6] Hisano [7] has also contributed.

Parallel to this development has been the extensive work in England on daylighting. It is based on the Waldram formula [8] (1923), which assumes that the window is a portion of a circular cylinder whose axis passes through the point P at which the pharosage is to be calculated. This assumption gives a simple equation and one that is sufficiently accurate for a single narrow window. But obviously the Waldram formula is worthless for a long rectangular source or for a long row of windows. Strangely enough, even the latest British publications continue to use the Waldram formula and to completely ignore [9] the correct equations which are employed by the rest of the world.

There are two limitations to all the work mentioned in the foregoing paragraphs:

(1) Attention is concentrated on conditions at a fixed point P. A visualization of the complete light distribution can be obtained only

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after a lengthy process which must be repeated for each change of the source.

(2) The results apply only to sources of uniform helios.[10] This is a desirable condition for artificial lighting but it does not apply to daylight. Actual skies are never of uniform helios. According to Knowles and Ives,[11] daylight calculations made on the assumption of uniform helios may be in error by a factor of 2 or 3.

The first investigation of the non-uniform rectangular window seems to have been made by Zee [12] who expressed the helios distribution by a power series. A later development [13] employed a cosine distribution of helios, which was found to approximate an average overcast sky. Methods were also developed for the ready visualization of the pharosage distribution in the room. The graphs,[13] however, applied only to the horizontal plane.

The present paper attempts to remove both of the limitations of previous investigations. New tables are given for uniform and non-uniform rectangular sources, and the results are also presented in the form of graphs. These data allow the determination of light distribution on horizontal, vertical, and tilted surfaces.

2. FUNDAMENTAL EQUATIONS.

According to the unit-sphere method,[14] the pharosage (lumen m.$^{-2}$) produced by a small source of helios H is

$$dD_N = \frac{H}{\pi} d\sigma'', \tag{1}$$

where $d\sigma''$ is the area cut from a unit sphere by the convergent lightcone and projected onto the illuminated surface. In Fig. 1, the illuminated surface has the normal PN' specified by the angles ξ and η. The lightcone has its apex at P and cuts from the unit sphere at Q the area

$$d\sigma' = \sin\theta \, d\theta \, d\varphi.$$

The projection of this area onto the illuminated plane gives the area

$$d\sigma'' = \sin\theta \cos\psi \, d\theta \, d\varphi.$$

But by spherical trigonometry,

$$\cos\psi = \cos\xi \cos\theta + \sin\xi \sin\theta \cos(\varphi - \eta).$$

Thus,

$$dD_N = \frac{H}{\pi} \sin\theta \left[\cos\xi \cos\theta + \sin\xi \sin\theta \cos(\varphi - \eta)\right] d\theta \, d\varphi. \tag{2}$$

Equation (2) applies to an infinitesimal lightcone. For a cone of finite extent, Eq. (2) is integrated, giving

$$D_N = \sin\xi \sin\eta \int\int \frac{H(\theta, \varphi)}{\pi} \sin^2\theta \sin\varphi \, d\theta \, d\varphi + \cos\xi \int\int \frac{H(\theta, \varphi)}{\pi} \sin\theta \cos\theta \, d\theta \, d\varphi + \sin\xi \cos\eta \int\int \frac{H(\theta, \varphi)}{\pi} \sin^2\theta \cos\varphi \, d\theta \, d\varphi. \quad (3)$$

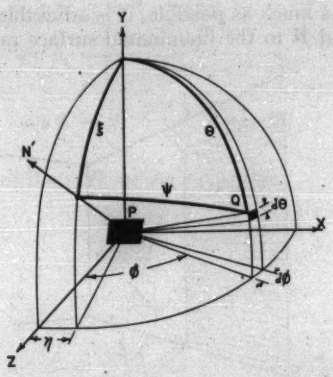


FIG. 1. Unit sphere and coordinate system.

Equation (3) is of great generality since it applies to any size and shape of source and any helios distribution. For the special case of an illuminated surface in the YZ-plane, $\xi = \eta = \pi/2$ and Eq. (3) reduces to

$$D_x = \frac{1}{\pi} \int\int H(\theta, \varphi) \sin^2\theta \sin\varphi \, d\theta \, d\varphi. \quad (4)$$

Similarly, for $\xi = 0$,

$$D_y = \frac{1}{\pi} \int\int H(\theta, \varphi) \sin\theta \cos\theta \, d\theta \, d\varphi \quad (5)$$

and for $\xi = \pi/2$, $\eta = 0$,

$$D_z = \frac{1}{\pi} \int\int H(\theta, \varphi) \sin^2\theta \cos\varphi \, d\theta \, d\varphi. \quad (6)$$

A knowledge of the three quantities D_x, D_y, and D_z allows one to obtain

the pharosage on a surface oriented in any manner. According to Eq. (3),

$$D_N = D_x \sin \xi \sin \eta + D_y \cos \xi + D_z \sin \xi \cos \eta. \qquad (3a)$$

The foregoing equations have been derived without the use of vector methods. Usually, however, it is convenient to think of the quantities D_x, D_y, and D_z as coordinates of the pharosage vector $\mathbf{D}$:

$$\mathbf{D} = \mathbf{i}D_x + \mathbf{j}D_y + \mathbf{k}D_z. \qquad (7)$$

The vector $\mathbf{D}$ is in the direction of energy flow [15] and is thus associated with the Poynting vector of electromagnetic theory. To avoid the use of negative signs as much as possible, it is advisable to employ the inward-drawn normal $\mathbf{N}$ to the illuminated surface rather than the out-

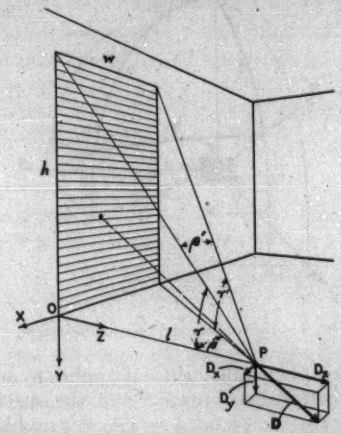


FIG. 2. Rectangular source and the pharosage vector it produces at point P.

ward-drawn normal N' of Fig. 1. The coordinate axes should also be drawn as in Fig. 2. The scalar pharosage (lumen m.$^{-2}$) on any tilted surface is then

$$D_N = \mathbf{D} \cdot \mathbf{N}, \qquad (8)$$

where $\mathbf{D}$ is obtained from Eq. (7). The vector method allows the quick determination of D_N from graphs for D_x, D_y, and D_z, as will be shown in Section 9.

3. UNIFORM RECTANGULAR SOURCES.

Consider a rectangular source of uniform helios H. The pharosage is to be obtained on a perpendicular erected at a corner of the source (Fig. 2). At point P, the angles subtended by the source are β and γ.

From Eqs. (4), (5), and (6),

$$D_x/H = \frac{1}{\pi}\int_0^{\beta}\int_{\theta_s}^{\pi/2} \sin^2\theta \sin\varphi \, d\theta \, d\varphi,$$

$$D_y/H = \frac{1}{\pi}\int_0^{\beta}\int_{\theta_s}^{\pi/2} \sin\theta \cos\theta \, d\theta \, d\varphi, \tag{9}$$

$$D_z/H = \frac{1}{\pi}\int_0^{\beta}\int_{\theta_s}^{\pi/2} \sin^2\theta \cos\varphi \, d\theta \, d\varphi,$$

where

$$\cot\theta_s = \tan\gamma\cos\varphi,$$

$$\sin\theta_s = \frac{\cos\gamma}{[1 - \sin^2\gamma\sin^2\varphi]^{\frac{1}{2}}}, \tag{10}$$

$$\cos\theta_s = \frac{\sin\gamma\cos\varphi}{[1 - \sin^2\gamma\sin^2\varphi]^{\frac{1}{2}}}.$$

Integration gives the well-known equations,

$$D_x/H = \frac{1}{2\pi}[\gamma - \gamma_1\cos\beta],$$

$$D_y/H = \frac{1}{2\pi}[\beta - \beta_1\cos\gamma], \tag{11}$$

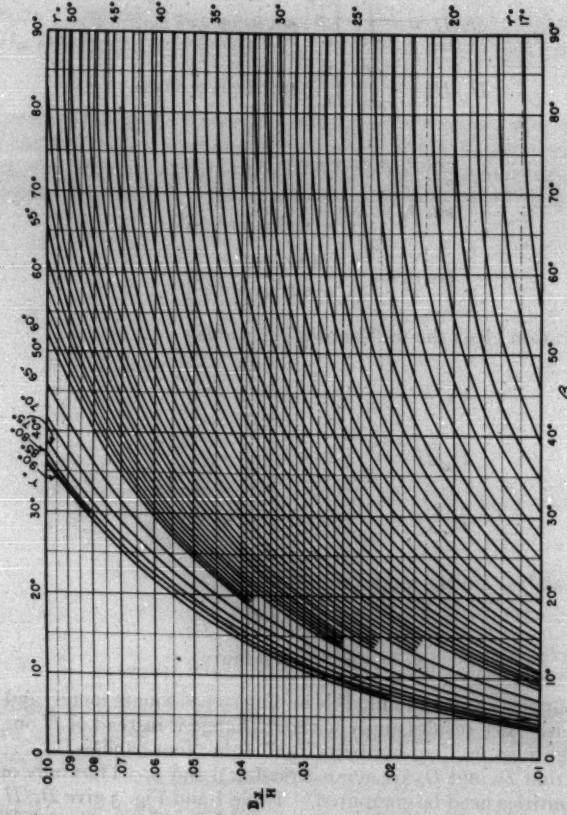
$$D_z/H = \frac{1}{2\pi}[\beta_1\sin\gamma + \gamma_1\sin\beta],$$

where β_1 and γ_1 are the angles shown in Fig. 2:

$$\beta_1 = \tan^{-1}(\tan\beta\cos\gamma),$$
$$\gamma_1 = \tan^{-1}(\cos\beta\tan\gamma). \tag{12}$$

Here the angles are in radians, D is in lumens per square meter, and H is in blondels. The results apply without change if instead of H one employs brightness in "foot-lamberts" and D in "foot-candles."

Note that D_x and D_y are symmetrical in β and γ. Thus only one of these quantities need be computed. Table I and Fig. 3 give D_y/H as a function of the two angles β and γ. Likewise Table II and Fig. 4 apply to D_z/H. D_x/H is obtained by interchanging β and γ in Table I and Fig. 3. The computations for Tables I and II were made to eight decimal places. All the results were checked by means of third differences and the numerical values were then rounded off to six decimal places. In general, the tables are believed to be correct in the sixth place.


90°
80°
70°
60°
50°
40°
30°
20°
10°
0
β
0.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
D/H

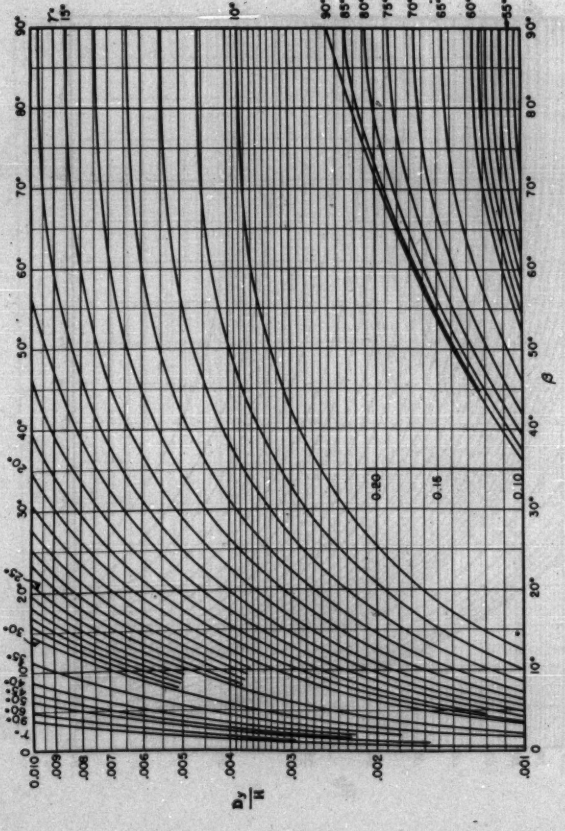
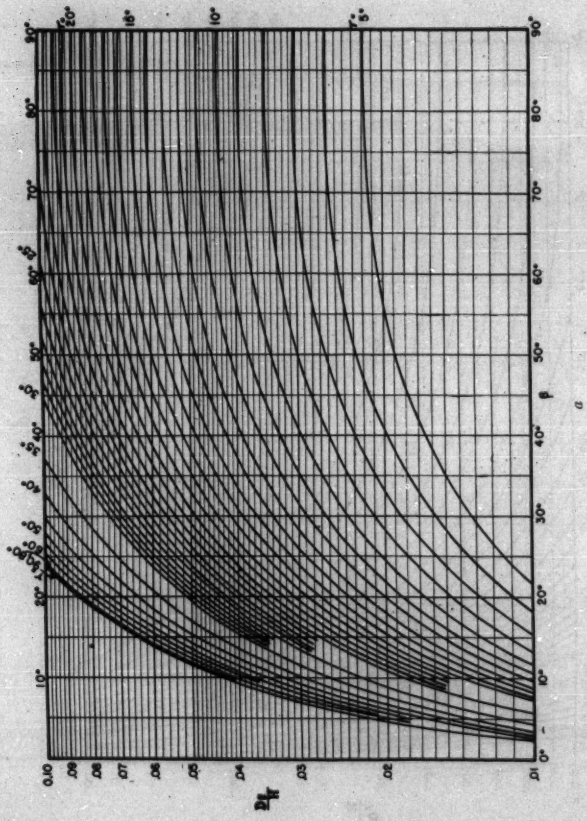


FIG. 3. Pharosage produced by a source of uniform helios. D_y/H as a function of angles.


90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
β
α
0.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
sin α/sin γ

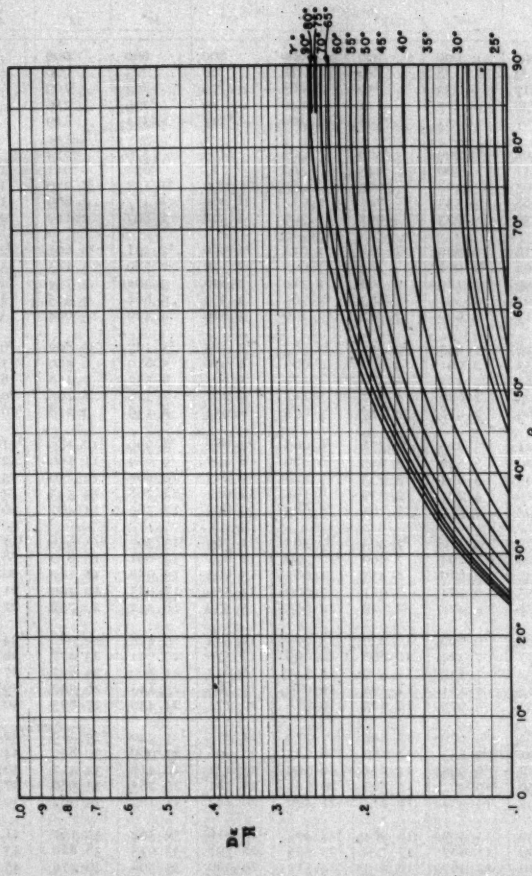


FIG. 4. Pharosage produced by a source of uniform helios. D_d/H as a function of angles.

TABLE I.

D_v/H for Uniform Rectangular Source.

γ (deg.)	$\beta=5°$	10°	15°	20°	25°	30°	35°	40°
0	000	000	000	000	000	000	000	000
1	$^{5}4$	$^{5}8$	$^{4}12$	$^{4}16$	$^{4}20$	$^{4}23$	$^{4}26$	$^{4}29$
2	$^{4}17$	$^{4}33$	$^{4}50$	$^{4}65$	$^{4}79$	$^{4}93$	$^{3}105$	$^{3}115$
3	$^{4}38$	$^{4}75$	$^{3}112$	$^{3}146$	$^{3}179$	$^{3}209$	$^{3}236$	$^{3}260$
4	$^{4}67$	$^{3}133$	$^{3}198$	$^{3}260$	$^{3}317$	$^{3}371$	$^{3}419$	$^{3}461$
5	$^{3}105$	$^{3}209$	$^{3}309$	$^{3}405$	$^{3}495$	$^{3}579$	$^{3}654$	$^{3}720$
6	151	300	445	583	713	$^{3}832$	$^{3}941$	$^{2}1$ 037
7	205	408	605	$^{3}793$	$^{3}969$	$^{2}1$ 132	$^{2}1$ 279	1 410
8	268	533	789	$^{2}1$ 034	$^{2}1$ 264	1 477	1 669	1 840
9	339	673	997	1 307	1 598	1 867	2 110	2 326
10	$^{3}418$	$^{3}829$	$^{2}1$ 229	$^{2}1$ 611	$^{2}1$ 970	$^{2}2$ 301	$^{2}2$ 601	$^{2}2$ 868
11	504	$^{2}1$ 001	1 484	1 945	2 379	2 780	3 143	3 465
12	599	1 189	1 762	2 310	2 826	3 302	3 734	4 118
13	701	1 392	2 063	2 705	3 309	3 868	4 375	4 826
14	811	1 610	2 387	3 130	3 829	4 476	5 064	5 587
15	$^{2}928$	$^{2}1$ 843	$^{2}2$ 732	$^{2}3$ 583	$^{2}4$ 385	$^{2}5$ 127	$^{2}5$ 802	$^{2}6$ 402
16	$^{2}1$ 053	2 091	3 100	4 065	4 976	5 819	6 586	7 270
17	1 184	2 353	3 488	4 576	5 601	6 552	7 418	8 189
18	1 323	2 628	3 897	5 114	6 261	7 326	8 295	$^{2}9$ 161
19	1 469	2 918	4 327	5 678	6 954	8 139	9 218	$^{1}10$ 182
20	$^{2}1$ 621	$^{2}3$ 220	$^{2}4$ 777	$^{2}6$ 269	$^{2}7$ 680	$^{2}8$ 990	$^{1}10$ 186	$^{1}11$ 254
21	1 780	3 536	5 245	6 886	8 437	$^{2}9$ 879	11 196	12 375
22	1 945	3 864	5 733	7 528	9 225	$^{1}10$ 805	12 250	13 543
23	2 116	4 204	6 239	8 194	$^{1}10$ 044	11 768	13 345	14 759
24	2 293	4 557	6 763	8 883	10 892	12 765	14 481	16 021
25	$^{2}2$ 475	$^{2}4$ 920	$^{2}7$ 303	$^{2}9$ 595	$^{1}11$ 769	$^{1}13$ 797	$^{1}15$ 656	$^{1}17$ 328
26	2 664	5 294	7 860	$^{1}10$ 330	12 672	14 861	16 870	18 678
27	2 857	5 679	8 433	11 085	13 603	15 958	18 122	20 072
28	3 055	6 074	9 021	11 860	14 559	17 085	19 409	21 507
29	3 258	6 478	$^{2}9$ 623	12 655	15 539	18 242	20 732	22 982
30	$^{2}3$ 466	$^{2}6$ 892	$^{1}10$ 239	$^{1}13$ 469	$^{1}16$ 543	$^{1}19$ 428	$^{1}22$ 088	$^{1}24$ 496
31	3 677	7 314	10 868	14 299	17 570	20 641	23 477	26 048
32	3 893	7 743	11 509	15 147	18 617	21 880	24 897	27 636
33	4 113	8 181	12 161	16 010	19 685	23 143	26 347	29 259
34	4 335	8 625	12 825	16 888	20 771	24 430	27 825	30 915
35	$^{2}4$ 562	$^{2}9$ 076	$^{1}13$ 498	$^{1}17$ 779	$^{1}21$ 875	$^{1}25$ 740	$^{1}29$ 329	$^{1}32$ 603
36	4 791	$^{2}9$ 533	14 180	18 684	22 996	27 069	30 859	34 321
37	5 022	$^{2}9$ 995	14 871	19 599	24 131	28 418	32 412	36 067
38	5 256	$^{1}10$ 462	15 569	20 525	25 281	29 785	33 987	37 840
39	5 492	10 934	16 274	21 460	26 442	31 167	35 583	39 639
40	$^{2}5$ 730	$^{1}11$ 409	$^{1}16$ 984	$^{1}22$ 404	$^{1}27$ 616	$^{1}32$ 564	$^{1}37$ 197	$^{1}41$ 460
41	5 969	11 887	17 700	23 355	28 798	33 975	38 829	43 302
42	6 210	12 367	18 419	24 312	29 990	35 396	40 474	45 164
43	6 451	12 850	19 142	25 274	31 188	36 828	42 133	47 043
44	6 693	13 334	19 867	26 239	32 392	38 267	43 804	48 938
45	$^{2}6$ 936	$^{1}13$ 818	$^{1}20$ 594	$^{1}27$ 207	$^{1}33$ 600	$^{1}39$ 713	$^{1}45$ 484	$^{1}50$ 846
46	7 178	14 303	31 321	28 176	34 811	41 164	47 171	52 765
47	7 420	14 787	22 048	29 146	36 023	42 618	48 864	54 693
48	7 662	15 271	22 774	30 115	37 235	44 073	50 561	56 628
49	7 902	15 752	23 497	31 081	38 445	45 527	52 259	58 567

Note—Superscripts refer to number of zeros after decimal point. Thus $^{3}1621 = 0.001621$.

TABLE I.—*Continued.*

γ (deg.)	$\beta = 5°$	10°	15°	20°	25°	30°	35°	40°
50	$^{2}8\ 142$	$^{1}16\ 232$	$^{1}24\ 218$	$^{1}32\ 044$	$^{1}39\ 652$	$^{1}46\ 979$	$^{1}53\ 956$	$^{1}60\ 508$
51	8 380	16 709	24 935	33 003	40 855	48 427	55 650	62 449
52	8 616	17 182	25 647	33 956	42 051	49 869	57 340	64 388
53	8 850	17 652	26 353	34 901	43 239	51 303	59 023	66 321
54	9 082	18 117	27 053	35 839	44 418	52 728	60 697	68 247
55	$^{2}9\ 312$	$^{1}18\ 577$	$^{1}27\ 746$	$^{1}36\ 768$	$^{1}45\ 587$	$^{1}54\ 141$	$^{1}62\ 359$	$^{1}70\ 163$
56	9 538	19 031	28 430	37 686	46 743	55 540	64 008	72 066
57	9 762	19 479	29 105	38 592	47 886	56 925	65 641	73 954
58	$^{2}9\ 982$	19 920	29 771	39 486	49 013	58 293	67 256	75 824
59	$^{1}10\ 198$	20 354	30 425	40 366	50 124	59 641	68 851	77 673
60	$^{1}10\ 410$	$^{1}20\ 780$	$^{1}31\ 068$	$^{1}41\ 230$	$^{1}51\ 216$	$^{1}60\ 969$	$^{1}70\ 423$	$^{1}79\ 499$
61	10 618	21 198	31 699	42 079	52 289	62 275	71 971	81 300
62	10 822	21 607	32 317	42 910	53 341	63 556	73 492	83 072
63	11 021	22 006	32 920	43 723	54 371	64 811	74 983	84 812
64	11 214	22 396	33 509	44 517	55 376	66 038	76 444	86 519
65	$^{1}11\ 403$	$^{1}22\ 775$	$^{1}34\ 082$	$^{1}45\ 290$	$^{1}56\ 357$	$^{1}67\ 236$	$^{1}77\ 870$	$^{1}88\ 189$
66	11 586	23 143	34 639	46 041	57 311	78 402	79 261	89 820
67	11 764	23 500	35 180	46 771	58 237	69 536	80 615	91 409
68	11 936	23 845	35 702	47 476	59 134	70 634	81 928	92 954
69	12 101	24 178	36 207	48 158	60 000	71 697	83 200	94 451
70	$^{1}12\ 261$	$^{1}24\ 499$	$^{1}36\ 692$	$^{1}48\ 814$	$^{1}60\ 835$	$^{1}72\ 721$	$^{1}84\ 427$	$^{1}95\ 898$
71	12 413	24 806	37 158	49 444	61 638	73 706	85 608	97 293
72	12 560	25 101	37 604	50 047	62 406	74 650	86 742	98 633
73	12 699	25 381	38 029	50 622	63 139	75 552	87 826	$^{1}99\ 916$
74	12 831	25 647	38 433	51 169	63 837	76 410	88 858	$^{0}101\ 140$
75	$^{1}12\ 956$	$^{1}25\ 899$	$^{1}38\ 815$	$^{1}51\ 687$	$^{1}64\ 497$	$^{1}77\ 223$	$^{1}89\ 837$	$^{0}102\ 302$
76	13 074	26 136	39 174	52 174	65 119	77 990	90 761	103 400
77	13 184	26 358	39 511	52 631	65 703	78 709	91 629	104 432
78	13 287	26 565	39 825	53 056	66 246	79 380	92 439	105 396
79	13 382	26 756	40 115	53 450	66 750	80 001	93 189	106 290
80	$^{1}13\ 469$	$^{1}26\ 932$	$^{1}40\ 382$	$^{1}53\ 811$	$^{1}67\ 211$	$^{1}80\ 572$	$^{1}93\ 878$	$^{0}107\ 112$
81	13 548	27 091	40 624	54 139	67 631	81 091	94 506	107 862
82	13 619	27 234	40 841	54 435	68 009	81 557	95 070	108 536
83	13 682	27 361	41 034	54 696	68 343	81 971	95 571	109 135
84	13 737	27 471	41 201	54 923	68 634	82 331	96 007	109 656
85	$^{1}13\ 783$	$^{1}27\ 565$	$^{1}41\ 343$	$^{1}55\ 116$	$^{1}68\ 881$	$^{1}82\ 636$	$^{1}96\ 377$	$^{0}110\ 098$
86	13 821	27 641	41 459	55 274	69 083	82 886	96 680	110 462
87	13 851	27 701	41 550	55 397	69 241	83 082	96 917	110 746
88	13 872	27 744	41 615	55 485	69 354	83 221	97 087	110 949
89	13 885	27 769	41 654	55 538	69 422	83 305	97 188	111 070
90	$^{1}13\ 889$	$^{1}27\ 778$	$^{1}41\ 667$	$^{1}55\ 556$	$^{1}69\ 444$	$^{1}83\ 333$	$^{1}97\ 222$	$^{0}111\ 111$

TABLE I.—*Continued.*

γ (deg.)	β=45°	50°	55°	60°	65°	70°	75°	80°
0	000	000	000	000	000	000	000	000
1	4 31	4 33	4 35	4 36	4 37	4 37	4 38	4 38
2	3 125	3 132	3 139	3 143	3 147	3 150	3 151	3 152
3	280	298	312	323	331	337	340	342
4	498	529	554	574	588	598	604	608
5	3 778	3 826	3 866	3 896	3 919	3 934	3 944	3 949
6	1 1 119	2 1 189	1 1 246	2 1 290	2 1 323	2 1 345	2 1 359	2 1 366
7	1 523	1 618	1 695	1 755	1 800	1 830	1 849	1 859
8	1 987	2 111	2 212	2 291	2 349	2 389	2 414	2 427
9	2 513	2 670	2 798	2 898	2 972	3 023	3 054	3 071
10	2 3 099	2 3 293	2 3 451	2 3 575	2 3 667	2 3 730	2 3 769	2 3 789
11	3 745	3 981	3 172	4 322	4 434	4 510	4 558	4 583
12	4 451	4 732	4 961	5 140	5 272	5 364	5 421	5 450
13	5 217	5 547	5 816	6 026	6 183	6 291	6 358	6 393
14	6 041	6 424	6 737	6 982	7 164	7 290	7 368	7 409
15	2 6 924	2 7 364	2 7 724	2 8 007	2 8 216	2 8 361	2 8 451	2 8 499
16	7 864	8 366	8 777	2 9 099	2 9 339	2 9 505	2 9 608	2 9 662
17	8 861	2 9 429	2 9 894	1 10 259	1 10 531	1 10 719	1 10 836	1 10 898
18	2 9 914	1 10 553	1 11 076	11 487	11 793	12 005	12 137	12 206
19	1 11 023	11 736	12 321	12 781	13 124	13 361	13 510	13 587
20	1 12 187	1 12 979	1 13 629	1 14 141	1 14 523	1 14 788	1 14 953	1 15 040
21	13 404	14 280	14 999	15 566	15 990	16 284	16 467	16 564
22	14 675	15 638	16 430	17 056	17 524	17 849	18 052	18 158
23	15 998	17 053	17 922	18 609	19 124	19 482	19 706	19 823
24	17 371	18 524	19 474	20 226	20 790	21 183	21 429	21 558
25	1 18 795	1 20 049	1 21 085	1 21 906	1 22 522	1 22 951	1 23 221	1 23 362
26	20 268	21 628	22 753	23 646	24 318	24 786	25 080	25 235
27	21 789	23 260	24 479	25 448	26 177	26 687	27 007	27 176
28	23 356	24 943	26 260	27 309	28 099	28 653	29 001	29 184
29	24 969	26 677	28 096	29 228	30 083	30 683	31 060	31 259
30	1 26 626	1 28 459	1 29 986	1 31 206	1 32 129	1 32 776	1 33 185	1 33 401
31	28 326	30 290	31 929	33 240	34 234	34 933	35 374	35 608
32	30 067	32 167	33 923	35 330	36 398	37 151	37 627	37 879
33	31 848	34 089	35 966	37 474	38 621	39 430	39 943	40 215
34	33 668	36 055	38 059	39 671	40 900	41 770	42 321	42 614
35	1 35 524	1 38 063	1 40 198	1 41 921	1 43 236	1 44 168	1 44 761	1 45 075
36	37 416	40 112	42 384	44 220	45 626	46 625	47 260	47 599
37	39 342	42 199	44 613	46 570	48 070	49 139	49 820	50 184
38	41 299	44 324	46 886	48 966	50 567	51 708	52 438	52 828
39	43 287	46 484	49 199	51 410	53 114	54 333	55 114	55 532
40	1 45 302	1 48 678	1 51 552	1 53 897	1 55 711	1 57 011	1 57 846	1 58 294
41	47 344	50 904	53 942	56 428	58 356	59 742	60 634	61 114
42	49 410	53 160	56 367	59 001	61 048	62 524	63 476	63 990
43	51 499	55 443	58 826	61 613	63 786	65 356	66 373	66 922
44	53 608	57 752	61 317	64 262	66 566	68 237	69 321	69 908
45	1 55 734	1 60 084	1 63 838	1 66 948	1 69 389	1 71 165	1 72 320	1 72 948
46	57 877	62 438	66 385	69 667	72 252	74 138	75 370	76 040
47	60 032	64 810	68 958	72 418	75 152	77 156	78 468	79 185
48	62 199	67 199	71 553	75 198	78 090	80 216	81 614	82 379
49	64 374	69 601	74 168	78 005	81 061	83 317	84 805	85 623

TABLE I.—*Continued.*

γ (deg.)	$\beta = 45°$	50°	55°	60°	65°	70°	75°	80°
50	166 556	172 014	176 801	180 837	184 064	186 456	188 041	188 915
51	68 740	74 436	79 448	83 691	87 097	89 633	91 320	92 254
52	70 926	76 864	82 107	86 563	90 157	92 845	94 641	95 639
53	73 110	79 295	84 775	89 453	93 242	96 090	198 001	199 068
54	75 289	81 725	87 449	92 356	96 349	99 365	0101 400	1102 540
55	177 461	184 152	190 126	195 269	199 475	0102 669	0104 834	0106 055
56	79 622	86 573	92 802	98 189	0102 617	105 998	108 304	109 609
57	81 770	88 984	95 475	0101 113	105 773	109 351	111 805	113 203
58	83 902	91 382	198 139	104 037	108 938	112 723	115 337	116 834
59	86 014	93 764	0100 793	106 958	112 109	116 113	118 897	120 501
60	188 104	196 125	0103 431	0109 870	0115 282	0119 517	0122 482	0124 202
61	90 168	198 464	106 050	112 771	118 454	122 931	126 089	127 936
62	92 204	0100 775	108 646	115 656	121 619	126 352	129 717	131 700
63	94 207	103 055	111 215	118 219	124 774	129 776	133 361	135 492
64	96 176	105 300	113 752	121 358	127 913	133 197	137 019	139 311
65	198 105	0107 507	0116 252	0124 165	0131 031	0136 611	0140 686	0143 154
66	199 993	109 671	118 712	126 937	134 124	140 014	144 358	147 018
67	0101 836	111 788	121 126	129 668	137 184	143 399	148 031	150 901
68	103 630	113 855	123 489	132 352	140 206	146 760	151 701	154 800
69	105 373	115 867	125 797	134 983	143 183	150 091	155 360	158 710
70	107 061	117 820	128 045	137 555	146 108	153 384	159 003	162 628
71	108 691	119 711	130 226	140 062	148 974	156 631	162 623	166 550
72	110 260	121 535	132 337	142 497	151 772	159 824	166 211	170 470
73	111 764	123 288	134 372	144 854	154 496	162 954	169 760	174 382
74	113 201	124 965	136 326	147 127	157 136	166 010	173 258	178 279
75	114 568	126 565	138 193	149 307	159 683	168 982	176 695	182 152
76	115 861	128 081	139 969	151 389	162 129	171 858	180 057	185 992
77	117 078	129 511	141 649	153 365	164 463	174 625	183 330	189 786
78	118 217	130 852	143 227	155 229	166 677	177 271	186 497	193 519
79	119 274	132 099	144 699	156 973	168 760	179 781	189 541	197 174
80	120 248	133 249	146 060	158 592	170 704	182 142	192 442	200 728
81	121 137	134 300	147 305	160 079	172 497	184 338	195 177	204 155
82	121 937	135 248	148 432	161 427	174 131	186 355	197 722	207 422
83	122 648	131 091	149 435	162 632	175 597	188 178	200 054	210 492
84	123 267	136 827	150 312	163 687	176 887	189 792	202 146	213 320
85	123 794	137 453	151 060	164 588	177 993	191 184	203 971	215 854
86	124 227	137 968	151 675	165 332	178 907	192 342	205 506	218 040
87	124 564	138 370	152 156	165 914	179 625	193 255	206 727	219 819
88	124 806	138 658	152 501	166 331	180 141	193 913	207 614	221 137
89	124 952	138 831	152 709	166 583	180 452	194 311	208 153	221 958
90	125 000	138 889	152 778	166 667	180 556	194 444	208 333	222 222

TABLE I.—*Continued.*

γ (deg.)	β=85°	90°	γ (deg.)	β=85°	90°	γ (deg.)	β=85°	90°
0	000	000	30	^{1}33 482	^{1}33 494	60	124 896	125 000
1	^{4}38	^{4}38	31	35 695	35 708	61	128 685	128 798
2	^{3}152	^{3}152	32	37 974	37 988	62	132 510	132 632
3	343	343	33	40 318	40 332	63	136 369	136 502
4	609	609	34	42 725	42 741	64	140 262	140 407
5	^{3}951	^{3}951	35	^{1}45 195	^{1}45 212	65	144 187	144 345
6	^{2}1 369	^{2}1 370	36	47 727	47 746	66	148 142	148 316
7	1 863	1 863	37	50 321	50 341	67	152 127	152 317
8	2 432	2 433	38	52 976	52 997	68	156 139	156 348
9	3 077	3 078	39	55 690	55 714	69	160 176	160 408
10	^{2}3 797	^{2}3 798	40	^{1}58 464	^{1}58 489	70	164 238	164 495
11	4 592	4 593	41	61 296	61 323	71	168 322	168 608
12	5 462	5 463	42	64 185	64 214	72	172 426	172 746
13	6 406	6 407	43	67 131	67 162	73	176 548	176 907
14	7 424	7 426	44	70 132	70 154	74	180 685	181 091
15	^{2}8 516	^{2}8 519	45	^{1}73 188	^{1}73 223	75	184 834	185 295
16	^{2}9 682	^{2}9 685	46	76 298	76 335	76	188 991	189 520
17	^{1}10 921	^{1}10 924	47	79 460	79 500	77	193 153	193 762
18	12 232	12 236	48	82 674	82 717	78	197 313	198 022
19	13 616	13 620	49	85 939	85 985	79	201 465	202 298
20	^{1}15 072	^{1}15 077	50	^{1}89 253	^{1}89 303	80	205 597	206 588
21	16 600	16 605	51	92 617	92 670	81	209 698	210 891
22	18 198	18 204	52	96 027	96 085	82	213 749	215 207
23	19 867	19 874	53	^{1}99 485	^{1}99 546	83	217 720	219 533
24	21 607	21 614	54	^{0}102 988	^{0}103 054	84	221 473	223 868
25	^{1}23 415	^{1}23 423	55	^{0}106 535	^{0}106 606	85	225 243	228 211
26	25 293	25 301	56	110 125	110 202	86	228 638	232 561
27	27 239	27 248	57	113 758	113 840	87	231 621	236 916
28	29 253	29 263	58	117 431	117 520	88	234 003	241 275
29	31 334	31 345	59	121 144	121 240	89	235 564	245 637
						90	236 111	250 000

TABLE II.

D_0/H for Uniform Rectangular Source.

γ (deg.)	$\beta=5°$	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
0	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000
5	^{2}2 412	^{2}4 787	^{2}7 091	^{2}9 290	^{2}11 354	^{2}13 258	^{2}14 980	^{2}16 504	^{2}17 821	^{2}18 928	^{2}19 827	^{2}20 529	^{2}21 047	^{2}21 404	^{2}21 625	^{2}21 740	^{2}21 783	^{2}21 789
10	4 787	^{2}9 503	^{2}14 079	^{2}18 450	22 556	26 347	29 779	32 822	35 454	37 669	39 471	40 878	41 919	42 637	43 081	43 313	43 400	43 412
15	7 091	^{2}14 079	20 865	27 353	33 457	39 100	44 220	48 768	52 711	56 037	58 748	60 870	62 443	63 529	64 203	64 555	64 686	64 705
20	^{2}9 290	18 450	27 353	35 877	43 912	51 358	58 130	64 162	69 409	73 847	77 478	80 327	^{2}82 446	^{2}83 912	^{2}84 824	^{2}85 301	^{2}85 479	^{2}85 505
25	^{2}11 354	22 556	33 457	43 912	53 789	62 968	71 343	78 831	^{2}85 368	^{2}90 921	^{2}95 482	^{2}99 075	^{2}101 759	^{2}103 622	^{2}104 784	^{2}105 394	^{2}105 622	^{2}105 655
30	13 258	26 347	39 100	51 358	62 968	73 792	83 706	^{2}92 607	^{2}100 416	^{2}107 082	^{2}112 585	^{2}116 944	120 215	122 496	123 925	124 678	124 959	125 000
35	14 980	29 779	44 220	58 130	71 343	83 706	^{2}95 078	^{2}105 339	114 390	122 162	128 619	133 766	137 652	140 379	142 096	143 004	143 345	143 394
40	16 504	32 822	48 768	64 162	78 831	^{2}92 607	^{2}105 339	116 889	127 140	136 002	143 419	149 378	153 913	157 118	159 150	160 230	160 638	160 697
45	17 821	35 454	52 711	69 409	85 368	^{2}100 416	114 390	127 140	138 532	148 456	156 833	163 624	168 843	172 566	174 947	176 223	176 707	176 777
50	18 928	37 669	56 037	73 847	90 921	107 082	122 162	136 002	148 456	159 396	168 718	176 357	182 294	186 580	189 354	190 853	191 428	191 511
55	19 827	39 471	58 748	77 478	95 482	112 585	128 619	143 419	156 833	168 718	178 951	187 438	194 124	199 024	202 241	204 006	204 688	204 788
60	20 529	40 878	60 870	80 327	^{2}99 075	116 944	133 766	149 378	163 624	176 357	187 438	196 748	204 201	209 762	213 487	215 569	216 386	216 506
65	21 047	41 919	62 443	82 446	^{2}101 759	120 215	137 652	153 913	168 843	182 294	194 124	204 201	212 410	218 668	222 967	225 436	226 428	226 577
70	21 404	42 637	63 529	83 912	103 622	122 496	140 379	157 118	172 566	186 580	199 024	209 762	218 668	225 626	230 562	233 506	234 734	234 923
75	21 625	43 081	64 203	84 824	104 784	123 925	142 096	159 150	174 947	189 354	202 241	213 487	222 967	230 562	236 153	239 668	241 229	241 482
80	^{2}21 740	^{2}43 313	^{2}64 555	^{2}85 301	105 394	124 678	143 004	160 230	176 223	190 853	204 006	215 569	225 436	233 506	239 668	243 803	245 836	246 202
85	21 783	43 400	64 686	85 479	105 622	124 959	143 345	160 638	176 707	191 428	204 688	216 386	226 428	234 734	241 229	245 836	248 444	249 024
90	21 789	43 412	64 705	85 505	105 655	125 000	143 394	160 697	176 777	191 511	204 788	216 506	226 577	234 923	241 482	246 202	249 024	250 000

4. UNIFORM SOURCES, D AS A FUNCTION OF DISTANCE.

In some applications, the foregoing tabulation in terms of β and γ is convenient. If scale drawings of the room are available, for instance, the angles β and γ can be read directly by means of a protractor and the pharosage can then be obtained from Figs. 3 or 4. A better picture of the pharosage distribution is obtained, however, by expressing the results in terms of the dimensions of the source and the distance to the point P. Let w = width of source, h = height of source, l = distance from plane of source to P. Since these distances will occur always in ratios, any unit of distance may be employed. From Fig. 2,

$$\begin{aligned}
\sin\beta &= \frac{w/h}{[(w/h)^2 + (l/h)^2]^{\frac{1}{2}}},\\
\cos\beta &= \frac{l/h}{[(w/h)^2 + (l/h)^2]^{\frac{1}{2}}},\\
\sin\gamma &= \frac{1}{[1 + (l/h)^2]^{\frac{1}{2}}},\\
\cos\gamma &= \frac{l/h}{[1 + (l/h)^2]^{\frac{1}{2}}},\\
\tan\beta_1 &= \frac{w/h}{[1 + (l/h)^2]^{\frac{1}{2}}},\\
\tan\gamma_1 &= \frac{1}{[(w/h)^2 + (l/h)^2]^{\frac{1}{2}}}.
\end{aligned} \tag{13}$$

TABLE III.

D_s/H for Uniform Source.

l/h	$w/h=\infty$	2.0	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$
0.10	23413	24046	22791	22173	21651	21263	20733	19982	18860	17049	13799	$^{1}73179$
0.15	22630	22080	21702	20789	20026	19465	18705	17513	16108	13753	$^{1}99695$	$^{1}41921$
0.20	21858	21128	20630	19438	18457	17742	16791	15494	13671	11046	$^{1}72821$	$^{1}26292$
0.30	20362	19278	18556	16869	15535	14595	13388	11823	$^{1}97902$	$^{1}72008$	$^{1}41444$	$^{1}12700$
0.40	18944	17521	16597	14521	12958	11899	10592	$^{1}89900$	$^{1}70601$	$^{1}48482$	$^{1}25688$	$^{2}72893$
0.60	16399	14357	13126	10595	$^{1}88993$	$^{1}78344$	$^{1}66404$	$^{1}52920$	$^{1}38711$	$^{1}24500$	$^{1}11981$	$^{2}31990$
0.80	14261	11692	10279	$^{1}76697$	$^{1}61157$	$^{1}52198$	$^{1}42612$	$^{1}32684$	$^{1}22888$	$^{1}13894$	$^{2}65540$	$^{2}17045$
1.0	12500	$^{1}95069$	$^{1}80292$	$^{1}55660$	$^{1}42607$	$^{1}35515$	$^{1}28266$	$^{1}21117$	$^{1}14415$	$^{2}85673$	$^{2}39566$	$^{2}10186$
1.5	$^{1}93583$	$^{1}57248$	$^{1}43914$	$^{1}26522$	$^{1}18912$	$^{1}15206$	$^{1}11676$	$^{2}84304$	$^{2}55768$	$^{2}32245$	$^{2}14642$	$^{2}36606$
2.0	$^{1}73792$	$^{1}35547$	$^{1}25345$	$^{1}13929$	$^{2}95668$	$^{2}75567$	$^{2}57073$	$^{2}40600$	$^{2}26499$	$^{2}15167$	$^{2}68436$	$^{2}17189$
3.0	$^{1}51208$	$^{1}15382$	$^{2}99615$	$^{2}49656$	$^{2}31910$	$^{2}25592$	$^{2}19051$	$^{2}13401$	$^{2}86421$	$^{2}49179$	$^{2}22122$	$^{3}55704$
4.0	$^{1}38990$	$^{2}76741$	$^{2}47237$	$^{2}22504$	$^{2}14722$	$^{2}11380$	$^{2}84034$	$^{2}59683$	$^{2}37879$	$^{2}20849$	$^{3}95493$	$^{3}23875$
6.0	$^{1}26284$	$^{2}26069$	$^{2}15342$	$^{2}70505$	$^{2}45518$	$^{2}35014$	$^{2}25942$	$^{2}17984$	$^{2}11618$	$^{3}65253$	$^{3}28909$	$^{3}72337$
8.0	$^{1}19792$	$^{2}11586$	$^{2}66845$	$^{2}30398$	$^{2}19735$	$^{2}15120$	$^{2}11141$	$^{3}77986$	$^{3}52521$	$^{3}27653$	$^{3}12300$	$^{3}30764$
10.0	$^{1}15863$	$^{2}60813$	$^{2}34855$	$^{2}15708$	$^{2}10055$	$^{3}91003$	$^{3}56818$	$^{3}38038$	$^{3}25306$	$^{3}14324$	$^{3}63219$	$^{3}15811$

Note: Superscripts refer to number of zeros after decimal point.

TABLE IV.

D_v/H for Uniform Source.

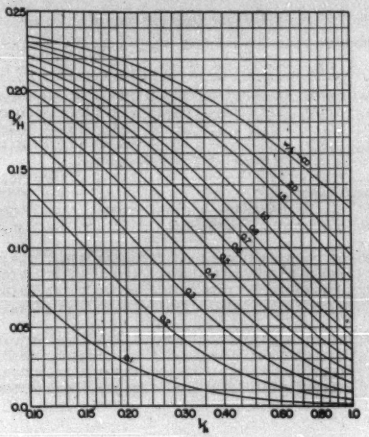
l/h	$w/h = \infty$	2.0	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000
0.10	22512	22454	22387	22173	21955	21779	21518	21128	20501	19418	17310	12343
0.15	21292	21204	21105	20789	20470	20209	19837	19275	18400	16939	14297	^{1}91256
0.20	20097	19983	19851	19438	19024	18693	18218	17521	16453	14748	11896	^{1}70741
0.30	17817	17647	17456	16860	16299	15854	15235	14357	13076	11220	^{1}84928	^{1}46841
0.40	15717	15494	15248	14462	13844	13330	12637	11692	10398	^{1}86071	^{1}62939	^{1}33518
0.60	12138	11861	11496	10595	^{1}98350	^{1}92951	^{1}86090	^{1}77405	^{1}66525	^{1}52726	^{1}37299	^{1}19282
0.80	^{1}93825	^{1}89889	^{1}86099	^{1}76693	^{1}69555	^{1}64669	^{1}58852	^{1}51894	^{1}43693	^{1}34221	^{1}23587	^{1}12043
1.0	^{1}73223	^{1}68690	^{1}64706	^{1}55736	^{1}49484	^{1}45475	^{1}40853	^{1}35547	^{1}29544	^{1}22862	^{1}15605	^{2}79181
1.5	^{1}41988	^{1}36715	^{1}33099	^{1}26518	^{1}22667	^{1}20454	^{1}18013	^{1}15381	^{1}12562	^{2}95795	^{2}64648	^{2}32566
2.0	^{1}26393	^{1}21123	^{1}18303	^{1}13928	^{1}11650	^{1}10394	^{2}90702	^{2}76744	^{2}62182	^{2}47110	^{2}31642	^{2}15890
3.0	^{1}12829	^{2}84336	^{2}69200	^{2}49656	^{2}40664	^{2}35921	^{2}29939	^{2}26085	^{2}22472	^{2}15815	^{2}10580	^{2}53014
4.0	^{2}74645	^{2}40584	^{2}32276	^{2}22504	^{2}18255	^{2}16075	^{2}13830	^{2}11570	^{2}93042	^{2}69964	^{3}46728	^{2}23396
6.0	^{2}34033	^{2}13385	^{2}10345	^{2}70665	^{2}56659	^{2}49815	^{2}42749	^{2}35682	^{2}28584	^{2}21470	^{2}14324	^{3}71620
8.0	^{2}19305	^{2}58728	^{2}44563	^{2}30080	^{2}24318	^{2}21310	^{2}18303	^{2}15247	^{2}12223	^{2}91514	^{3}61274	^{3}30558
10.0	^{2}12407	^{2}30876	^{2}23396	^{2}15661	^{2}12541	^{2}10982	^{3}94379	^{3}78622	^{3}62866	^{3}47269	^{3}31493	^{3}15758

TABLE V.

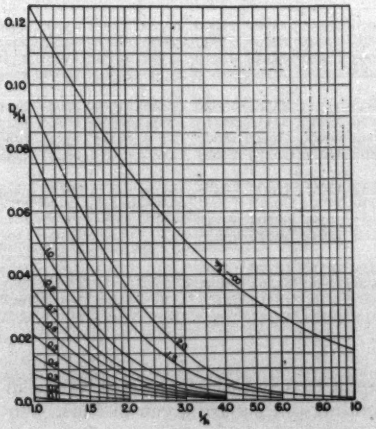
D_a/H for Uniform Source.

l/h	$w/h = \infty$	20	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000	^{0}25000
0.10	24876	24865	24849	24796	24737	24686	24602	24465	24214	23685	22340	17666
0.15	24723	24693	24662	24546	24419	24306	24130	23961	23319	22293	20015	13844
0.20	24515	24468	24408	24209	23988	23790	23501	23026	22208	20684	17599	11140
0.30	23946	23838	23711	23288	22839	22460	21900	21040	19703	17429	13697	^{1}78196
0.40	23212	23026	22810	22116	21413	20844	20039	18866	17139	14586	10900	^{1}59217
0.60	21437	21042	20609	19347	18207	17370	16243	14777	12862	10411	^{1}73854	^{1}38472
0.80	19522	18866	18206	16479	15085	14128	12945	11497	^{1}97485	^{1}76865	^{1}53269	^{1}27297
1.0	17678	16737	15876	13860	12382	11436	10320	^{1}90185	^{1}75256	^{1}58454	^{1}40019	^{1}20343
1.5	13868	12236	11094	^{1}89418	^{1}76697	^{1}69266	^{1}61128	^{1}52270	^{1}42742	^{1}32631	^{1}22036	^{1}11106
2.0	11180	^{1}90183	^{1}78392	^{1}59863	^{1}50143	^{1}44776	^{1}39084	^{1}33090	^{1}26827	^{1}20332	^{1}13660	^{2}68621
3.0	^{1}79057	^{1}52266	^{1}42914	^{1}30828	^{1}25275	^{1}22315	^{1}19296	^{1}16204	^{1}13045	^{2}98321	^{2}65782	^{2}32959
4.0	^{1}60633	^{1}33623	^{1}26318	^{1}18370	^{1}14901	^{1}13115	^{1}11301	^{2}94574	^{2}75941	^{2}57119	^{2}38156	^{2}19104
6.0	^{1}41100	^{1}16336	^{1}12514	^{2}85264	^{2}68653	^{2}60237	^{2}51751	^{2}43212	^{2}34627	^{2}26004	^{2}17346	^{2}86808
8.0	^{1}31008	^{2}94586	^{2}72167	^{2}48719	^{2}39119	^{2}34282	^{2}29424	^{2}24548	^{2}19681	^{2}14753	^{2}98405	^{3}49217
10.0	^{1}24876	^{2}61618	^{2}46734	^{2}31412	^{2}24500	^{2}21826	^{2}18927	^{2}15785	^{2}12635	^{2}94805	^{3}63224	^{3}31544

Calculated values in terms of dimensions are listed in Tables III to V, and the corresponding graphs are given in Figs. 5 to 7. In this case, unlike Tables I and II, calculations were made to only five digits, all of which were retained in the final tables. Thus some errors may be expected in the fourth and fifth figures.

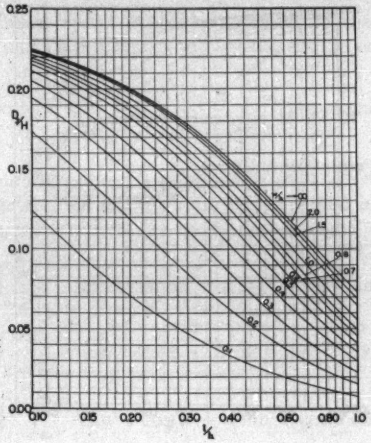


a

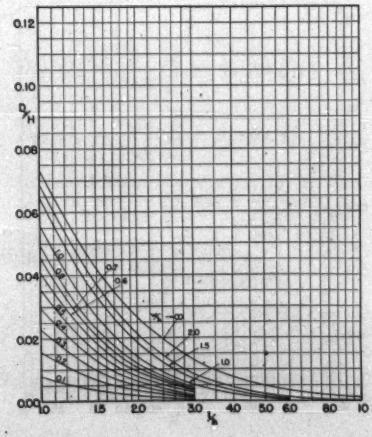


b

FIG. 5. D_s/H as a function of dimensions. Uniform helios.

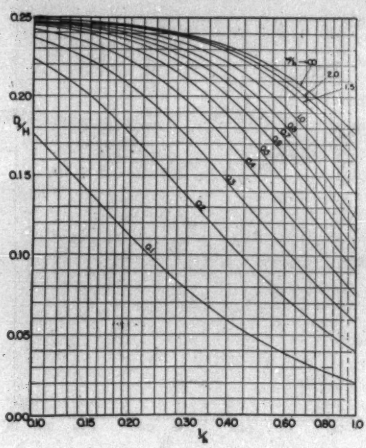


a

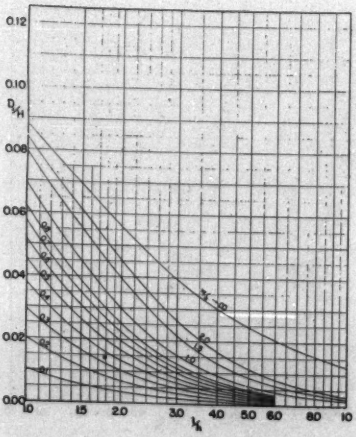


b

FIG. 6. D_v/H as a function of dimensions. Uniform helios.



a



b

FIG. 7. D_s/H as a function of dimensions. Uniform helios.

5. NON-UNIFORM SOURCES.

It has been shown [13] that experimental data on the helios variation in overcast skies can be approximated by the simple equation,

$$H(\theta) = \frac{3}{7} D_u (1 + 2 \cos \theta), \tag{14}$$

where

$H(\theta)$ = helios (blondel) at zenith angle θ,
D_u = pharosage (lumen m.$^{-2}$) on an unobstructed horizontal surface outdoors. Equations (4) to (6) now become

$$D_x/D_u = \frac{3}{7\pi} \int_0^{\beta} \int_{\theta_x}^{\pi/2} (\sin^2 \theta \sin \varphi + 2 \sin^2 \theta \cos \theta \sin \varphi)\, d\theta\, d\varphi,$$

$$D_y/D_u = \frac{3}{7\pi} \int_0^{\beta} \int_{\theta_y}^{\pi/2} (\sin \theta \cos \theta + 2 \sin \theta \cos^2 \theta)\, d\theta\, d\varphi, \tag{15}$$

$$D_z/D_u = \frac{3}{7\pi} \int_0^{\beta} \int_{\theta_z}^{\pi/2} (\sin^2 \theta \cos \varphi + 2 \sin^2 \theta \cos \theta \cos \varphi)\, d\theta\, d\varphi.$$

If we write

$$D/D_u = D_0/D_u + D_1/D_u, \tag{16}$$

then from Section 4,

$$D_{x0}/D_u = \frac{3}{14\pi} (\gamma - \gamma_1 \cos \beta),$$

$$D_{y0}/D_u = \frac{3}{14\pi} (\beta - \beta_1 \cos \gamma), \tag{17}$$

$$D_{z0}/D_u = \frac{3}{14\pi} (\beta_1 \sin \gamma + \gamma_1 \sin \beta).$$

Also, by integration,

$$D_{x1}/D_u = \frac{2}{7\pi} [(1 - \cos \gamma) - \cos \beta (1 - \cos \gamma_1)],$$

$$D_{y1}/D_u = \frac{2}{7\pi} [\zeta - \sin \zeta \cos \gamma \cos \gamma_1] \tag{18}$$

$$D_{z1}/D_u = \frac{2}{7\pi} \sin \beta (1 - \cos^2 \gamma \cos \gamma_1)$$

where

$$\sin \zeta = \sin \beta \sin \gamma. \tag{19}$$

These equations can also be written in terms of the dimensions of the source. The uniform component, Eq. (17), is obtained by use of the previous results, Tables III to V, multiplied by 3/7 to take into account the different coefficients of Eqs. (11) and (17). The cosine component, Eq. (18), is found by substituting Eq. (13) in (18). The actual equations employed in the calculations are

$$D_{x1}/D_u = \frac{2}{7\pi}\left[\left(1 - \frac{(l/h)}{[1+(l/h)^2]^{\frac{1}{2}}}\right) - \frac{(l/h)}{[(w/h)^2+(l/h)^2]^{\frac{1}{2}}}\left(1 - \left[\frac{(w/h)^2+(l/h)^2}{1+(w/h)^2+(l/h)^2}\right]^{\frac{1}{2}}\right)\right],$$

$$D_{y1}/D_u = \frac{2}{7\pi}\left[\zeta - \frac{(w/h)(l/h)}{[1+(l/h)^2][1+(w/h)^2+(l/h)^2]^{\frac{1}{2}}}\right], \qquad (20)$$

$$D_{z1}/D_u = \frac{2}{7\pi}\frac{(w/h)}{[(w/h)^2+(l/h)^2]^{\frac{1}{2}}} \times\left[1 - \frac{(l/h)^2}{[1+(l/h)^2]}\left[\frac{(w/h)^2+(l/h)^2}{1+(w/h)^2+(l/h)^2}\right]^{\frac{1}{2}}\right].$$

Also

$$\sin\zeta = \frac{(w/h)}{[(1+(l/h)^2)((w/h)^2+(l/h)^2)]^{\frac{1}{2}}}$$

or

$$\tan\zeta = \frac{(w/h)}{(l/h)[1+(w/h)^2+(l/h)^2]^{\frac{1}{2}}}. \qquad (21)$$

TABLE VI.

D_z/D_u for Non-Uniform Source.

l/h	$w/h=\infty$	2.0	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$
0.10	18224	18018	17856	17429	17049	16759	16357	15780	14907	13487	10924	$^{1}57955$
0.15	17444	17137	16896	16265	15710	15291	14717	13845	12710	10866	$^{1}78890$	$^{1}33176$
0.20	16679	16271	15953	15131	14419	13887	13168	12232	10763	$^{1}87115$	$^{1}57512$	$^{1}20785$
0.30	15208	14604	14143	12984	12020	11324	10417	$^{1}92259$	$^{1}76732$	$^{1}56480$	$^{1}32570$	$^{1}99920$
0.40	13836	13044	12456	11064	$^{1}99145$	$^{1}91362$	$^{1}81622$	$^{1}69519$	$^{1}54781$	$^{1}37727$	$^{1}20041$	$^{1}56958$
0.60	11444	10312	$^{1}95349$	$^{1}78292$	$^{1}66310$	$^{1}58663$	$^{1}49911$	$^{1}39972$	$^{1}29334$	$^{1}18627$	$^{1}91285$	$^{1}24414$
0.80	$^{1}95253$	$^{1}81107$	$^{1}72275$	$^{1}54965$	$^{1}44219$	$^{1}37910$	$^{1}31085$	$^{1}23942$	$^{1}16825$	$^{1}10246$	$^{1}48439$	$^{1}12634$
1.0	$^{1}80210$	$^{1}63837$	$^{1}54717$	$^{1}38046$	$^{1}29821$	$^{1}24989$	$^{1}19968$	$^{1}14971$	$^{1}10242$	$^{2}61071$	$^{2}28307$	$^{2}72937$
1.5	$^{1}55382$	$^{1}35906$	$^{1}27954$	$^{1}17142$	$^{1}12301$	$^{2}99044$	$^{2}76367$	$^{2}55238$	$^{2}36642$	$^{2}21231$	$^{2}96402$	$^{2}23964$
2.0	$^{1}41228$	$^{1}21157$	$^{1}15259$	$^{2}84858$	$^{2}58500$	$^{2}46336$	$^{2}35017$	$^{2}24970$	$^{2}16322$	$^{2}93833$	$^{2}42244$	$^{2}10732$
3.0	$^{1}26613$	$^{2}85097$	$^{2}55442$	$^{2}26790$	$^{2}18026$	$^{2}14338$	$^{2}10703$	$^{2}75066$	$^{2}48760$	$^{2}27898$	$^{2}12420$	$^{2}28056$
4.0	$^{1}19425$	$^{2}40439$	$^{2}24969$	$^{2}11954$	$^{2}77717$	$^{2}59683$	$^{2}45027$	$^{2}30817$	$^{2}20190$	$^{3}11036$	$^{3}50777$	$^{3}12700$
6.0	$^{1}12503$	$^{2}12959$	$^{2}76449$	$^{2}35610$	$^{2}22693$	$^{2}17225$	$^{2}12846$	$^{2}90718$	$^{2}58251$	$^{3}32529$	$^{3}14421$	$^{3}36080$
8.0	$^{2}91849$	$^{2}55831$	$^{2}31926$	$^{2}14853$	$^{2}94905$	$^{2}72726$	$^{2}53540$	$^{2}48616$	$^{2}24482$	$^{3}13317$	$^{3}59232$	$^{3}14815$
10.0	$^{2}72498$	$^{2}28036$	$^{2}15824$	$^{2}75262$	$^{2}47371$	$^{2}42282$	$^{2}26765$	$^{2}17981$	$^{2}12382$	$^{3}67443$	$^{3}29975$	$^{3}74509$

TABLE VII.

D_s/D_u for Non-Uniform Source.

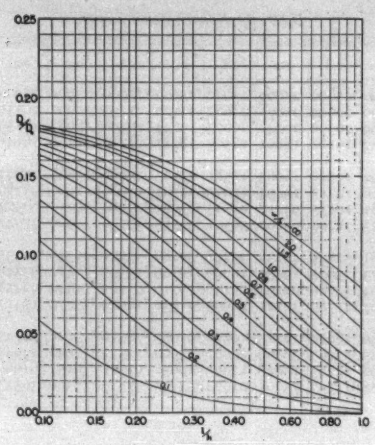
l/h	$w/h=\infty$	2.0	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$	$^{0}25000$
0.10	22126	22090	22042	21871	21687	21529	21293	20931	20336	19290	17222	12298
0.15	20721	20669	20596	20346	20073	19843	19498	18987	18159	16752	14159	$^{1}90603$
0.20	19354	19282	19188	18861	18509	18217	17790	17146	16141	14506	11729	$^{1}69896$
0.30	16768	16662	16526	16063	15581	15193	14638	13836	12643	10885	$^{1}82630$	$^{1}45674$
0.40	14425	14286	14113	13494	12975	12531	11919	11067	$^{1}98780$	$^{1}82210$	$^{1}60190$	$^{1}32121$
0.60	10560	10381	10135	$^{1}94417$	$^{1}88203$	$^{1}83664$	$^{1}77784$	$^{1}70211$	$^{1}60574$	$^{1}48393$	$^{1}34176$	$^{1}17699$
0.80	$^{1}77338$	$^{1}74923$	$^{1}72289$	$^{1}65202$	$^{1}59501$	$^{1}55555$	$^{1}50746$	$^{1}44909$	$^{1}37942$	$^{1}29806$	$^{1}20592$	$^{1}10529$
1.0	$^{1}57335$	$^{1}54581$	$^{1}51849$	$^{1}45252$	$^{1}40441$	$^{1}37289$	$^{1}33611$	$^{1}29339$	$^{1}24455$	$^{1}18974$	$^{1}12973$	$^{2}65901$
1.5	$^{1}29494$	$^{1}26373$	$^{1}23995$	$^{1}19450$	$^{1}16706$	$^{1}15109$	$^{1}13337$	$^{1}11411$	$^{1}93348$	$^{1}71288$	$^{1}48155$	$^{2}24275$
2.0	$^{1}17100$	$^{1}14064$	$^{1}12286$	$^{2}94294$	$^{2}79140$	$^{2}70719$	$^{2}61789$	$^{2}52344$	$^{2}42458$	$^{2}32191$	$^{2}21635$	$^{2}10869$
3.0	$^{2}74762$	$^{2}50667$	$^{2}41780$	$^{2}30121$	$^{2}24699$	$^{2}21836$	$^{2}18679$	$^{2}15873$	$^{2}13413$	$^{2}96298$	$^{2}64568$	$^{2}32310$
4.0	$^{2}40838$	$^{2}22840$	$^{2}18218$	$^{2}12725$	$^{2}10340$	$^{2}91083$	$^{2}78378$	$^{2}65628$	$^{2}52743$	$^{2}39670$	$^{2}26492$	$^{2}13115$
6.0	$^{2}17308$	$^{2}69723$	$^{2}53967$	$^{2}36869$	$^{2}29602$	$^{2}25960$	$^{2}22332$	$^{2}18648$	$^{2}14942$	$^{2}11218$	$^{2}74885$	$^{2}37433$
8.0	$^{2}94372$	$^{2}29298$	$^{2}22263$	$^{2}15028$	$^{2}12095$	$^{2}10652$	$^{2}91418$	$^{2}76157$	$^{2}61060$	$^{2}45722$	$^{2}30616$	$^{2}15288$
10.0	$^{2}59101$	$^{2}14988$	$^{2}11355$	$^{4}75995$	$^{4}61074$	$^{4}53345$	$^{4}45823$	$^{4}38360$	$^{4}30535$	$^{4}22968$	$^{4}15316$	$^{4}80584$

TABLE VIII.

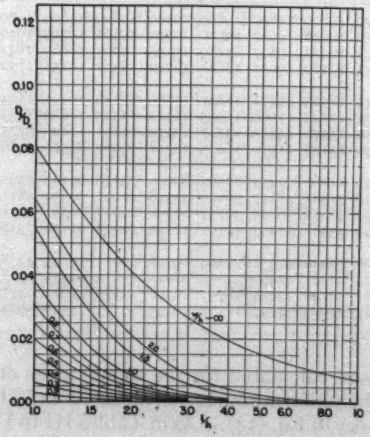
D_s/D_u for Non-Uniform Source.

l/h	$w/h=\infty$	2.0	1.5	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.00	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$	$^{0}19809$
0.10	19666	19658	19648	19612	19570	19531	19468	19363	19166	18753	17691	13993
0.15	19490	19473	19453	19373	19279	19195	19062	18889	18435	17631	15809	10958
0.20	19251	19243	19186	19052	18888	18742	18522	18158	17524	16333	13906	$^{1}88072$
0.30	18607	18544	18525	18171	17847	17566	17146	16491	15449	13700	10773	$^{1}61555$
0.40	17788	17682	17544	17060	16563	16145	15545	14659	13339	11371	$^{1}85094$	$^{1}46276$
0.60	15874	15649	15376	14523	13717	13110	12291	11206	$^{1}97758$	$^{1}79292$	$^{1}56342$	$^{1}29381$
0.80	13912	13540	13128	11980	11015	10341	$^{1}94987$	$^{1}84565$	$^{1}71868$	$^{1}56779$	$^{1}39410$	$^{1}20216$
1.0	12124	11594	11062	$^{1}97454$	$^{1}87502$	$^{1}80990$	$^{1}73258$	$^{1}64165$	$^{1}53655$	$^{1}41748$	$^{1}28618$	$^{1}14560$
1.5	$^{1}87417$	$^{1}78428$	$^{1}71581$	$^{1}58227$	50128	$^{1}45358$	$^{1}40091$	$^{1}34334$	$^{1}28114$	$^{1}21485$	$^{1}14521$	$^{2}73222$
2.0	$^{1}66103$	$^{1}54454$	$^{1}47632$	$^{1}36624$	30757	$^{1}27496$	$^{1}24027$	$^{1}20361$	$^{1}16521$	$^{1}12529$	$^{2}84218$	$^{2}42318$
3.0	$^{1}42977$	$^{1}29094$	$^{1}23986$	$^{1}17293$	14191	$^{1}12539$	$^{1}10928$	$^{2}91136$	$^{2}73386$	$^{2}55326$	$^{2}37066$	$^{2}18553$
4.0	$^{1}31335$	$^{1}17726$	$^{1}13949$	$^{2}97554$	$^{2}79194$	$^{2}69721$	$^{2}60074$	$^{2}50299$	$^{2}40388$	$^{2}30383$	$^{2}20297$	$^{2}10099$
6.0	$^{1}20072$	$^{2}81223$	$^{2}62341$	$^{2}42499$	$^{2}34234$	$^{2}30045$	$^{2}25812$	$^{2}21553$	$^{2}17273$	$^{2}12971$	$^{2}86544$	$^{2}43296$
8.0	$^{1}14688$	$^{2}45502$	$^{2}34740$	$^{2}23454$	$^{2}18797$	$^{2}16508$	$^{2}14170$	$^{2}11822$	$^{2}94778$	$^{2}71048$	$^{2}47394$	$^{2}23712$
10.0	$^{1}11561$	$^{2}29027$	$^{2}22021$	$^{2}14797$	$^{2}11572$	$^{2}10295$	$^{2}89183$	$^{2}74398$	$^{2}59534$	$^{2}44687$	$^{2}29797$	$^{2}14377$

The values were calculated for the same values of (w/h) and (l/h) employed with the uniform source. Tables VI to VIII give results for the non-uniform sky of Eq. (14). As in Tables III to V, the calculations were made to only five figures. The data are plotted in Figs. 8 to 10. Results apply, without change if H is replaced by brightness in equivalent lumens/m.^{2} and D is in lumens/m.^{2}, or if "foot-lamberts" and "foot-candles" are used.

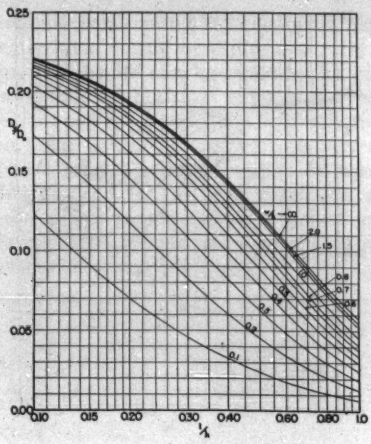


a

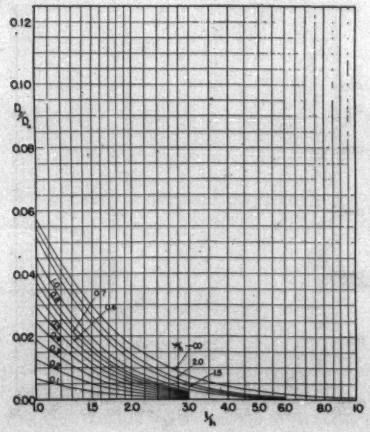


b

FIG. 8. D_z/D_o for an overcast sky.

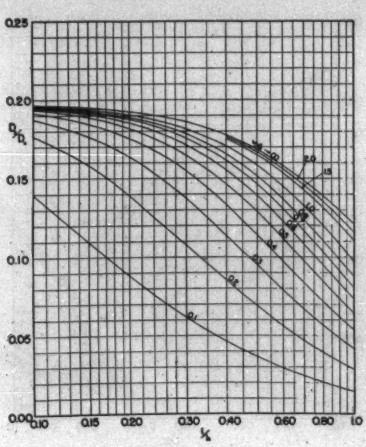


a

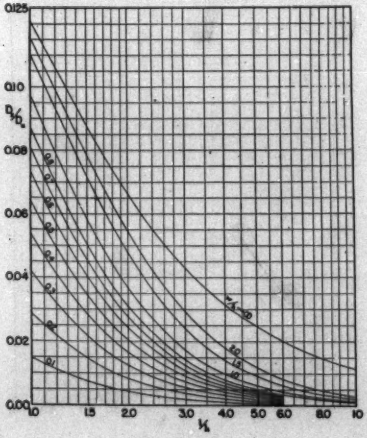


b

FIG. 9. D_u/D_z for an overcast sky.



a



b

FIG. 10. D_s/D_u for an overcast sky.

6. SUPERPOSITION.

All of the foregoing treatment has been restricted to points on a perpendicular erected at a corner of the source. By use of fictitious windows, however, the results can be extended to give pharosage at any point. This method of fictitious sources can be applied in the usual way [6] to the data of Figs. 3 and 4. The process must be repeated, however, for each point P and no idea can be gained of the over-all distribution without a large amount of work. It is here that the new method of plotting (Figs. 5 to 10) is of particular advantage. The linear vertical scale allows superposition of curves graphically, while the logarithmic horizontal scale enables a change of window height to be effected by shifting the curve sideways.

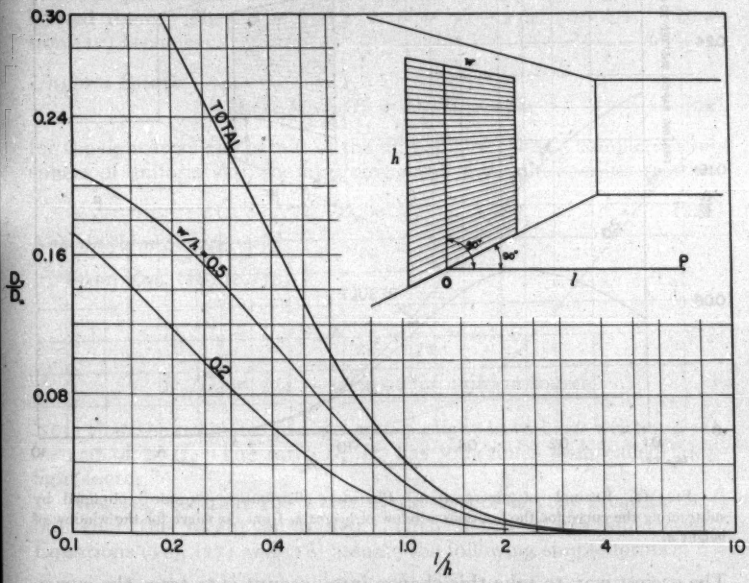


FIG. 11. Example of superposition. Pharosage distribution ("total") obtained by adding the ordinates of the curves marked "0.5" and "0.2."

Suppose that the pharosage distribution is needed along the line OP of Fig. 11. The actual source is considered to be divided into two parts, one to the left of O and the other to the right. A superposition of the curves for these two windows gives the resulting pharosage distribution along OP. For example, suppose that $w/h = 0.2$ for the left source while $w/h = 0.5$ for the right. The graphical addition of the

ordinates of these two curves (Fig. 9), by means of a pair of dividers, gives the upper curve of Fig. 11.

The other type of superposition is illustrated in Fig. 12. Two fictitious sources are now used, one of height h_1 and the other of height h_2. The actual pharosage distribution is obtained by subtracting the distribution curves associated with the two fictitious sources. Suppose that $w/h_1 = 0.5$ and $w/h_2 = 2.0$. The curve for w/h_1 is obtained directly from Fig. 9. The curve for $w/h_2 = 2.0$ cannot be used immediately, however, since it is referred to a different base ($h_2 = h_1/4$).

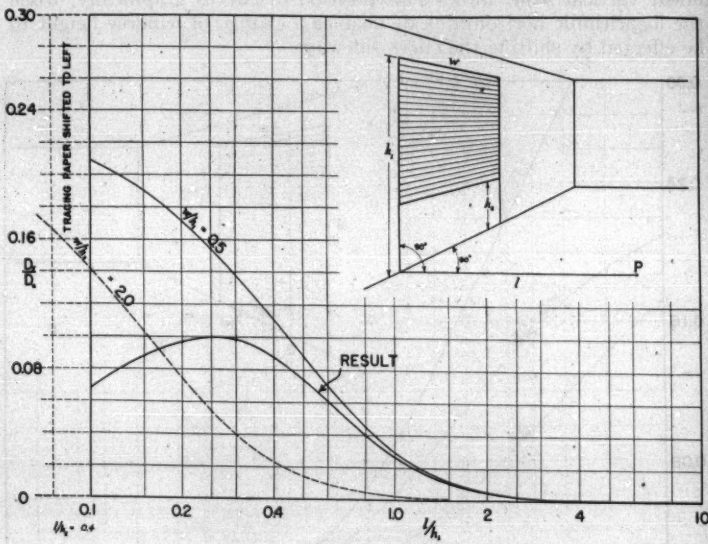


FIG. 12. Example of superposition. Pharosage distribution ("result") obtained by subtracting the curve for the fictitious window of height h_2 from the curve for the window of height h_1.

The easiest way to take this change into account is to trace the curve, $w/h_2 = 2.0$, on a piece of tracing paper and slide it to the left on Fig. 9 until the line marked $l/h_2 = 0.4$ coincides with the line $l/h_1 = 0.1$ of the original graph. The two curves are then subtracted graphically, giving the curve marked "result" in Fig. 12. This graphical method can be used by any architect in designing fenestration to produce satisfactory daylighting and it can be employed by any illuminating engineer in deciding on the size and placement of luminous panels. It is equally applicable to uniform and non-uniform sources by use of the

proper curves, Figs. 5 to 7 or 8 to 10. An obvious combination of the methods of Figs. 11 and 12 allows the determination of pharosage anywhere.

7. SPECIAL CASES.

The general equations for the pharosage produced by a rectangular source are too complicated for everyday engineering calculation. Hence the foregoing tables and graphs. But special cases are of frequent occurrence in which the general formulas reduce to very simple expressions.

First consider what happens if the source is very large compared with the distance l. For instance, the pharosage is to be calculated near the corner of a window or in the corner of a very large indirectly lighted room. Then $\beta = \gamma \to \pi/2$, $\beta' = \gamma' \to \pi/4$, $\zeta \to \pi/2$. Equation (11) becomes

Uniform Source

$$D_z/H = D_y/H = D_x/H = 1/4 \tag{22}$$

for the light from one octant of the unit sphere. For a complete hemisphere of uniform sky, the foregoing result gives the familiar relation,

$$D_y = H. \tag{22a}$$

Non-uniform Source

From Eqs. (16) to (18),

$$D_z/D_u = D_x/D_u = \frac{3}{28} + \frac{2}{7\pi} = 0.198, \tag{23}$$

$$D_y/D_u = 1/4 = \text{same as for uniform source.}$$

Note that the ordinates of Figs. 3 to 7 might have been written D/D_u instead of D/H. The ratio D_y/D_u has sometimes been called "daylight factor."

Now suppose that the width of the source is very great ($w \gg l$) but that the height is limited. Then $\beta \cong \beta' \cong \pi/2$ and $\gamma' \cong 0$. Equations (11), (17) and (18) assume the following simple forms:

Uniform Source

$$D_z/H \cong \frac{\gamma}{2\pi},$$

$$D_y/H \cong \frac{1}{4}(1 - \cos\gamma), \tag{24}$$

$$D_x/H \cong \frac{1}{4}\sin\gamma.$$

Non-uniform Source

$$D_x/D_u \leqq \frac{1}{14\pi}[3\gamma + 4(1 - \cos\gamma)],$$

$$D_y/D_u \leqq \frac{1}{14\pi}\left[\frac{3\pi}{2}(1 - \cos\gamma) + 4(\gamma - \sin\gamma\cos\gamma)\right], \qquad (25)$$

$$D_z/D_u \leqq \frac{1}{14\pi}\left[\frac{3\pi}{2}\sin\gamma + 4\sin^2\gamma\right].$$

The graphs of Figs. 5 to 10 are drawn for $l/h \leqq 10$. For greater distances, the inverse-square relation is applicable if $\beta \ll 1$ and $\gamma \ll 1$. Replacing the trigonometric functions of Eqs. (11) to (18) by their series representations and dropping the higher-power terms, one obtains

Uniform Source

$$D_x/H \leqq \frac{\beta^2\gamma}{2\pi} \leqq \frac{w^2h}{2\pi l^3},$$

$$D_y/H \leqq \frac{\beta\gamma^2}{2\pi} \leqq \frac{w^2h}{2\pi l^3}, \qquad (26)$$

$$D_z/H \leqq \frac{\beta\gamma}{\pi} \leqq \frac{wh}{\pi l^2}.$$

Non-uniform Source

$$D_x/D_u \leqq \frac{\beta^2\gamma}{14\pi}[3 + \gamma] \leqq \frac{w^2h}{14\pi l^3}[3 + h/l],$$

$$D_y/D_u \leqq \frac{\beta\gamma^2}{14\pi}[3 + 4\gamma] \leqq \frac{wh^2}{14\pi l^3}[3 + 4h/l], \qquad (27)$$

$$D_z/D_u \leqq \frac{3\beta\gamma}{7\pi}[1 + \gamma] \leqq \frac{3wh}{7\pi l^2}[1 + h/l].$$

All angles are expressed in radians.

8. TILTED SURFACES.

Pharosage has been evaluated for three particular surfaces, these results being designated as D_x, D_y, and D_z. The tables and graphs have a much wider applicability than this, however, for *they can be used in finding the pharosage on any surface*, as indicated in Section 2.

The pharosage vector $\mathbf{D}$ at point P, produced by a small light source at O, has the direction OP and a magnitude that varies inversely as the square of the distance. To find the scalar pharosage D_N (lumen m.$^{-2}$) on any surface at P, one merely employs the cosine relation:

$$D_N = \mathbf{D}\cdot\mathbf{N} = |\mathbf{D}|\cos\psi, \qquad (28)$$

where **N** is a unit vector pointing into the surface and normal to it, and ψ is the angle between **D** and **N**.

This use of the cosine relation with small sources is familiar to everyone. But the fact that the same relation is applicable to large sources, no matter what their size or shape, is not so well known. The rectangular source produces at every point in the lightfield a pharosage vector **D** just as the small source did. The only difference is that the direction of this vector is not obvious, though it is easily found by calculation. The pharosage vector is equal to the vector sum of its components, according to Eq. (7). The magnitude of this vector is

$$|\mathbf{D}| = [(D_x)^2 + (D_y)^2 + (D_z)^2]^{\frac{1}{2}}$$

and the pharosage D_N on any tilted surface is obtained by multiplying this magnitude by $\cos\psi$ in accordance with Eq. (28).

Better yet,

$$D_N = D_x \cos(\mathbf{i}, \mathbf{N}) + D_y \cos(\mathbf{j}, \mathbf{N}) + D_z \cos(\mathbf{k}, \mathbf{N}).$$

For instance, consider a tilted drafting board lighted by a window, Fig. 13. The values of D_x, D_y, and D_z are obtained from Figs. 8 to 10.

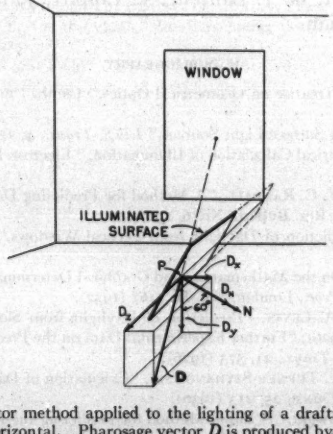


FIG. 13. The vector method applied to the lighting of a drafting board that makes an angle of 60° with the horizontal. Pharosage vector D is produced by the window. Projection of this vector on the inward-drawn normal N to the illuminated surface gives D_N, the pharosage (lumen m^{-2}) on the upper surface of the board.

The pharosage on the board is then

$$\begin{aligned} D_N &= D_x \cos 30° + D_y \cos 60° + D_z \cos 90° \\ &= 0.866\, D_x + 0.500\, D_y. \end{aligned}$$

The only caution that should be mentioned regarding the vector method is that the plane of the illuminated surface should not cut the source.[15] If it does cut the source, the result will be the *difference* in values of pharosage on the two sides of the illuminated surface instead of the value on one side.

9. SUMMARY.

The rectangular light source is more frequently encountered in engineering practice than any other form. Thus, both for artificial lighting and natural lighting, it is desirable to have convenient methods of obtaining the light distributions produced by rectangular sources. We have attempted to supply this need by calculating pharosage from sources of uniform helios and also for the non-uniform sources representing average overcast skies. Figures 5 to 10 are found to be particularly convenient in the visualization of light distributions.

The results of these graphs are readily extended to any point and to surfaces tilted at any angle, provided that the plane of the illuminated surface does not cut the source.

In conclusion, we wish to express our appreciation of the careful work of the computers: Messrs. L. S. Castlemann, M. S. Cettei, J. C. Gibbs, A. L. Keller, G. J. Laurent, V. Lippitt, J. M. Margolskee, L. Mautner, J. Topalian.

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Felling Trees Electrically.—"Burn 'em down with hot wires," is the advice of H. W. Bousman, General Electric engineer, when it comes to felling trees or pruning jobs about your property.

Mr. Bousman recently felled a large tree by the "hot-wire" method in his back yard at 104 Collins Street, Scotia. And the project was so successful that he plans to eliminate another undesirable tree on his property by the same method.

"It was a simple task," he says. "I had a large black locust tree which needed to be eliminated in order to improve my landscaping, but it was too near the house to be chopped down—not to mention the danger involved for such an inexperienced lumberjack.

"Neighbors had told me that professionals had rather high charges to fell trees of the same size on their property, so I rigged up a contraption to burn them down with hot wires. And it worked wonderfully."

Mr. Bousman felled the tree in sections, first burning off the upper branches, then parts of the 10-inch trunk.

Nichrome wire, which can be made "red hot" and still maintain its mechanical strength while being pulled through the timber, was attached in a semi-circle to the branches. A transformer was then connected with the wires in order to get high current at low voltage, he said, and then it was just a matter of "turning on the juice."

Weights were used to pull the burning wires through the wood.

While Mr. Bousman allowed the small branches to fall freely as the hot wire cut its way through, he explained that it was necessary to tie ropes around the trunk sections and then to adjoining trees in order to have them fall without damaging his home or other trees.

"The hot-wire system," he said, "does the job accurately and at the same time eliminates all personal danger to the worker."

Burning off the small limbs was a rather rapid process, but it required all night for the wires to cut through the trunk, the diameter of which was approximately 10 inches.

"As soon as I am able to purchase a transformer," said Bousman, "I plan to fell other trees on my property, as well as lend the apparatus to some neighbors who wish to fell trees after witnessing my demonstration."

Mr. Bousman said he also has another useful idea for his device. This winter he plans to use the hot-wire system for thawing the ice from around his garage doors.

"I expect to make a permanent installation with some resistant alloy at the base of the doors," he said. "Then when we have a freeze and the garage doors stick, as they did many times last winter, all I will have to do is turn on the hot-wire system for a neat thawing job."

R. H. O

THE PYRAMID OF PHYSICS.

BY

M. HERZBERGER.

Communication No. 1063 from the Kodak Research Laboratories.

Extremely rapid progress has been made in the field of physics in the last twenty years. It would be a most inappropriate time to question the basic concepts of theoretical physics, if it were not that the leading physicists feel that the present dualistic theory of the nature of matter and energy has certain contradictory aspects, and is highly unsatisfactory from a philosophical point of view. Both matter and energy seem to be of both corpuscular and wave nature: some physical phenomena can be explained by either aspect, while others can apparently be explained only by one or the other.

It seems to me that these contradictions arise, not from the physical laws which are so abundantly verified by experimental evidence, but from the simplified models with which we try to represent the true nature of things.

The Greek philosophers believed in a simple model of nature; such a model being a set of basic concepts corresponding to physical entities, from which logical conclusions might be drawn and verified by experiment. It is still our naive belief that such a system *must* exist; we still sin against the Second Commandment. The desirability of having some such models for physical phenomena is unquestioned, but we must never forget Kant's warning that the real object, the "*Ding an sich*," is unapproachable: we perceive it only through an instrument. We must always remember that we experience the image of the object and not the object itself, and that the image is, at best, a modification of the object by an instrument.

The fundamental premise presented in this paper is that the idea be abandoned, at least for the time being, that it is possible to find a system of simple mathematical concepts—a unified field theory—to explain all physical phenomena. Instead, it is suggested that a theory be built up in the form of a pyramid, in different levels, in such a way that the laws of one level will become the laws of the next lower level if a certain small but finite quantity can be neglected. Each physical problem could then be treated immediately in the simplest possible terms by assigning it to its own proper level.

As an illustration, let us consider a simple example. If we throw a pebble into the middle of a still pond, a system of waves emanates from the point of impact and soon forms a pattern over the entire surface.

Scientist A is interested in the wave pattern: the sizes, positions, and velocities of the various waves. He needs to know only the point of impact of the stone, its mass, and its velocity. The size, shape, and density of the stone are totally irrelevant to him.

Scientist B, on the other hand, wishes to understand what happens at the moment of impact. He needs to know the size, shape, and density of the stone, as well as its velocity. Still, and this is important, Scientist B knows no more than Scientist A about the wave pattern near the far shore of the pond, since at a sufficient distance from the stone, the differences in the wave pattern arising from differences in size or shape of the stone have all been smoothed out.

Pursuing the illustration still further, Scientist B need not inform himself about the molecular structure of either stone or water. His scientific object is not the ultimate truth, since he knows that stone and water are made up of molecules, the molecules of atoms, and that the atoms are complex solar systems in themselves.

The choice of the individual levels is quite arbitrary, and I am rather critical of my own choice, since the quantities neglected between the various levels have different dimensions.[1] The present subdivision was chosen so as to have the scheme correspond as closely as possible to the historical development and to avoid too radical a departure from traditional methods.

According to this choice, four different levels can be distinguished. The first level consists of purely geometrical constructions, in which there are curves (paths) and wave surfaces, or wave-surface elements in a suitable space. For optical phenomena, the space has three dimensions;[2] for mechanical and thermodynamic phenomena,[3] the space has $3n + 1$ dimensions, where n is the number of degrees of freedom available.

Since my special field is optics, I have tried to reduce the problem in the various fields of physics to their optical analogy, in a space of suitable dimensions. To distinguish this space from other spaces used in physical theory, for instance, phase space, the name *optical space* is suggested. Its properties are discussed below.

On the second level, the notion of wave length (λ) is introduced, and the problems of diffraction, interference, and polarization are discussed. On this level is also introduced the concept of aperture, or,

[1] *Dimension* is here used in the physical sense. Specifically, the quantities neglected are the wave length λ, having the dimension of length, $1/ct$, a reciprocal length, and the Planck constant h, the dimension of which is mc^2t.

[2] In this case the word *dimension* is used in the geometrical sense; that is, to denote the number of independent variables.

[3] The fact that optical and mechanical problems are equivalent was first shown by W. R. Hamilton. The fact that problems in all branches of physics may be reduced to the same variation problem is clearly expressed in Planck's system of physics.

in general, boundary conditions. I should like to point out that many of the ideas presented in quantum mechanics could well be handled on this level. The laws of the second level become the laws of the first level if it is permissible to set λ equal to zero.

Only stationary phenomena are considered on the second level. That is, all laws are derived on the assumption that energy transport is instantaneous. On the third level we take cognizance of the fact that energy travels with a great but finite velocity (c), and we shall investigate those phenomena for which this correction makes a measurable difference in the results. Here is introduced special relativity with all its consequences. On this level we have the general electromagnetic theory of energy and matter, and we must take into account the atomic structure of matter. The laws of this level are transformed to those of the second level by making the assumption that $1/ct$ is equal to zero.

The new phenomenon of the fourth level is the fact that the absorption of energy can be measured only in finite steps. The basis of the laws of this level is the Einstein effect, and the connection between frequency, wave length, and the Planck constant. They become laws of the third level if h is set equal to zero.

The advantages of arranging physical phenomena according to some such scheme are twofold. First, unnecessary mathematical complexity would be avoided by placing each problem immediately on the proper level, and undoubtedly many simplifications could be achieved at once in various fields of physics. Second, it would lead to more stability in the historical development. That is, the discovery of new facts would not mean a "revolution in our knowledge," but would lead only to the establishment of a new higher level and to the discovery of a quantity which has hitherto been neglected. The phenomena of the present highest level would then be much better understood, because their limitations and approximations would then be defined.

In developing these ideas, I hope to explain and justify my philosophical objections to the applications to physical problems of the theory of probability, especially the so-called Gaussian distribution. The use of statistical methods will be consistently permitted, inasmuch as they can be derived independently of the probability theory. I believe that we can give a causal description of physical phenomena if the concept of causality is modified slightly. It is true that only the image and not the object may be experienced. According to the theory here presented, however, the real object may be replaced by a simplified object, from which it cannot be distinguished. From the simplified object the image may be calculated and thus a causal picture of nature obtained. Statements about the object are only approximate, but the approximation is made with a finite and knowable accuracy.

It would obviously be absurd to try to derive all the laws of physics in this short paper; only a few of the more important problems can be

analyzed in any detail. The development of these problems is made to resemble as much as possible methods already familiar to physicists.

The models presented at several points may or may not be of value to the reader. Their value, or lack of it, does not affect the validity of the basic premises of the paper.

THE FIRST LEVEL.

One way of describing the first level is as follows:

In a homogeneous isotropic medium, the rays of light originating from a point are straight lines, and spheres having their centers at the luminous point intersect all the lines at right angles. If the light is reflected at a surface, or refracted into another homogeneous isotropic medium, its direction is changed, but it still travels in a series of straight lines normal to a series of parallel surfaces. These surfaces, however, are in general no longer spherical. In an inhomogeneous medium, the rays are no longer straight, but there is still through each point a wave surface which intersects all rays at right angles. In an anisotropic medium, the rays are straight lines and the wave surfaces are present, but they no longer intersect the system of rays normally.

One way of stating these facts mathematically, in the most general way, is the assumption of a function μ—the refractive index—which is given for a point in space as a function both of the position of the point and of the direction of the ray passing through the point.

The coordinates of the object point will be denoted by x_i, and the direction cosines of a ray by $\dot{x}_i = dx_i/ds$, with

$$\Sigma \dot{x}_i^2 = 1. \tag{1}$$

Using this relation, the function μ can be made homogeneous in the first order in the $\dot{x}_i$'s, so that it fulfills Euler's equation, namely,

$$\Sigma \dot{x}_i \frac{\partial \mu}{\partial \dot{x}_i} = \mu. \tag{2}$$

The curves we are interested in, the light rays, are then the extremals of the variation problem given by the equation,

$$\delta E = \delta \int \mu ds = 0, \tag{3}$$

and the wave-surface elements are perpendicular to the contravariant vector having the coordinates,

$$n^i = \frac{\partial \mu}{\partial \dot{x}_i}; \tag{4}$$

or, the wave-surface elements are perpendicular to the direction of the rays.

It should be noted that these equations have intentionally been written in such a form that they may be applied to a space of more than three dimensions.

The space in which a physical problem is located by a function having the qualities just described will be called the *optical space*, or *Hamiltonian space*, in honor of W. R. Hamilton, who was the first to make this transformation for mechanical problems.

It can be proved mathematically that any variation problem of the form

$$\delta E = \delta \int \tau dt, \tag{5}$$

where τ may still be dependent on t, can be transformed to the form discussed, by the simple expedient of transferring it to a space having one more dimension and considering t to be one of the independent variables.

For curves originating at a single point, the wave-surface elements can be arranged in wave surfaces, such that there is a vector $\vec{n}$ at each point perpendicular to the surface passing through that point.

Application to Mechanical Problems. A given mechanical system has n degrees of freedom. Then the state of the system is given by these n coordinates and by the time t which has elapsed since the initial moment. If t is considered as the $(n + 1)$th coordinate, the state of the system at any time can then be given as a function of the $n + 1$ coordinates x_i, and a change of state by a curve $x_i(s)$ in Hamiltonian space. To each state, that is, to each point of a curve, there belongs a momentum vector, having the coordinates n^i, which is contragradient to the vector $\dot{x}_i$. Hamilton then proved the existence of a function H, the difference between kinetic energy L and potential energy Φ, such that the curve $x_i(s)$ is an extremal for the variation problem given by H, and he also proved that the momentum vector can be calculated from the equations

$$n^i = \frac{\partial H}{\partial \dot{x}_i}, \qquad \Sigma \dot{x}_i \frac{\partial H}{\partial \dot{x}_i} = H. \tag{6}$$

The curves are given by the relation

$$\delta \int H ds = 0. \tag{7}$$

For the case of a potential function, the wave surfaces coincide with the potential surfaces. In electrodynamics, hydrodynamics, and aerodynamics we can consider the paths of many particles simultaneously, in agreement with the optical picture.

The laws are the same for statistical mechanics, but with the added complication that the number of degrees of freedom becomes very great.

Application to Thermodynamics. The entropy S is given by the formula

$$S = \int \frac{1}{T}(dU + pdV), \tag{8}$$

where T is the absolute temperature, p is the pressure, V is the volume, and U is the energy. A change of state takes place in such a way that S is a maximum for small changes, or

$$\delta S = 0. \tag{9}$$

We introduce independent parameters $q_1 \cdots q_n$, $q_{n+1} = t$, so that

$$\begin{aligned} dU &= U_\nu dq_\nu, \\ dV &= V_\nu dq_\nu. \end{aligned} \tag{10}$$

From this we find that

$$S = \int \frac{1}{T}(U_\nu + pV_\nu)dq_\nu. \tag{11}$$

Here the quantities $(U_\nu + pV_\nu)/T$ are the coordinates of the momentum vector $\vec{n}$ and the q's are the coordinates of the vector of position $\vec{a}$. The quantity s is the length of arc in the space whose dimensions are the q's. The "refractive index" μ may then be introduced, since

$$S = \int \vec{n}d\vec{a} = \int \mu ds, \tag{12}$$

where

$$\mu = \vec{n}\frac{d\vec{a}}{ds} = \Sigma \frac{1}{T}(U_\nu + pV_\nu)\frac{dq_\nu}{ds}.$$

These expressions may be interpreted by using the principles of statistical mechanics, and a problem in thermodynamics is thus transformed to one in mechanics.

Application to Electromagnetic Problems. Electromagnetic problems will be discussed in more detail on the higher levels. Here, it may suffice to say that the transport of all kinds of waves—heat rays, x-rays, gamma rays, etc.—follows the same laws that apply to waves of visible light, only with different values of the refractive index.

Some of the other qualities of optical space may be enumerated:

According to Fermat's principle, the light rays are the extremals of the variation principle given by the expression

$$E = \int \mu ds = \int n^i dx_i. \tag{13}$$

They are also the solution of Euler's differential equation

$$\frac{d}{ds}\left(\frac{\partial \mu}{\partial \dot{x}_i}\right) = \frac{\partial \mu}{\partial \dot{x}_i}, \tag{14}$$

which may be expanded into a system of second-order differential equations, namely,

$$\dot{x}_k \frac{\partial^2 \mu}{\partial \dot{x}_i \partial x_k} + \ddot{x}_k \frac{\partial^2 \mu}{\partial x_i \partial \dot{x}_k} = \frac{\partial \mu}{\partial \dot{x}_i}. \tag{15}$$

Reciprocally, any system of second-order differential equations which can be expressed in this form determines an optical system.

Another important theorem on this level, from which may be derived all the qualities of optical space, is that, for any two-dimensional manifold of rays whose parameters are u and v,

$$\frac{d}{ds}\left[\frac{\partial \vec{a}}{\partial u}\frac{\partial \vec{n}}{\partial v} - \frac{\partial \vec{a}}{\partial v}\frac{\partial \vec{n}}{\partial u}\right] = \frac{d}{ds} J = 0. \tag{16}$$

This is the well-known Lagrangean bracket, which has many applications in mathematical theory. Equation (16) gives the fundamental differential invariant of the problem in question. Since the number of possible parameters is $2n$, there are altogether $n(2n-1)$ such equations. These are the components of a generalized vector (or antisymmetrical tensor) which remains constant along a light ray. The integral invariant corresponding to equation (16) is the energy flux through a surface element. Equation (16) is the generalization of the differential operation called *rotation*, or *curl*, in three-dimensional vector analysis. Equation (16) shows therefore that the rotation remains constant along the path of a ray.

If a bundle of rays emanates from a point, we see immediately that $\partial \vec{a}/\partial u = \partial \vec{a}/\partial v = 0$ at that point. Therefore, we have the law that, for a bundle of rays meeting at a point,

$$\text{rot } (\vec{n}) = 0. \tag{17}$$

It is well known that this equation specified the existence of wave surfaces, or surfaces normal to $\vec{n}$ at each of their points.

So far, we have assumed μ to be continuous and continuously differentiable. If we now allow for surfaces separating two media, that is, surfaces of discontinuity separating spaces for which μ is continuous, we can derive the laws of refraction and reflection.

THE SECOND LEVEL.

Let us consider again the rays coming from a point. They have a system of wave surfaces, and each wave surface has a constant optical

distance ($\int \mu ds$) from the object point measured along the rays. From equation (17) it follows that

$$\vec{n} = \text{grad } \phi, \tag{18}$$

i.e., a potential function exists. Of importance is the value of the quantity

$$\Delta\phi = \text{div grad } \phi. \tag{19}$$

Then, if the radiation is restricted to a single wave length, the function ϕ fulfills the condition that

$$\Delta\phi = -k^2\phi, \qquad k = 2\pi/\lambda, \tag{20}$$

which means that ϕ as a function of its coordinates is a solution of the wave equation.

It then follows that ϕ repeats itself periodically throughout space in such a way that wave surfaces having a fixed optical distance λ have the same value of ϕ. (Actually, this corresponds to a Fourier analysis of ϕ.) The quantity λ is then called the wave length of the radiation.

This introduction of wave length involves no difficulties as long as there are well-defined wave surfaces, but what happens if some of the light is removed by introducing a finite aperture?

The answer to this question was first given by Fresnel. To find the solution we assume that the state of the light behind the barrier is still defined by a function ϕ, which is a solution of equation (20), the wave equation.

We know from mathematical analysis that the value of such a function can be determined for any point in a space, if its value is known on an enclosing surface. This condition is fulfilled if we assign to ϕ the value zero for the opaque parts of the screen and its undisturbed value over the transmitting aperture. The actual calculation of ϕ in an isotropic image space has been given by R. Luneberg,[4] who improved on Kirchhoff's method which is in general use in physics textbooks.

Kirchhoff assumed not only a knowledge of ϕ but also a knowledge of $d\phi/dz$ on the plane where $z = 0$. This is actually an over-determination, which is mathematically not permissible. Luneberg's improved formula requires only a knowledge of ϕ over the aperture plane.

In the simplest case of a perfect optical system with a circular aperture, the calculation of ϕ results in a light distribution of the image plane which is described by a Bessel function. Instead of a luminous point image, we see a little disk (the Airy disk) surrounded by concentric rings of rapidly decreasing intensity. The presence of lens aberrations changes the form of the central disk but, because of diffraction, we find that the image of a point is always a small surface element. In the simplest case, the diameter of the Airy disk is equal to $1.22\lambda/A$, and

[4] R. Luneberg, "Mathematical Theory of Optics," Brown University, 1944.

A, the numerical aperture of the optical system, is always less than unity. And even an aperture of unity produces an image that has a finite area.

From this we can conclude that only a finite number of "points" can be seen in an image, since two points whose apparent separation is small compared with the size of the Airy disk will not be resolved by the instrument. In particular, we can say that the instrument images a single point if the object surface element is so small that its image cannot be distinguished from an Airy disk. This is the case, for instance, with the image of a star.

This conclusion is of great importance to physicists. A luminous point is a mathematical fiction which cannot exist in physics, but a luminous surface element can exist. The fact that such an element can be considered a point as long as its image cannot be distinguished from an Airy disk gives a physical meaning to the expression "luminous point."

In this paper, therefore, the word *point* will henceforth refer to a surface element whose image cannot be distinguished from an Airy disk.

The problems of interference and diffraction can be studied on this second level. The phenomena of polarization may be added if ϕ is not required to be a scalar quantity, but may be a vector in the wave-surface element.

At this point we can introduce the conception of coherent and incoherent light, that is, light which does or does not demonstrate interference phenomena. Coherence can be defined very simply in the following way: Light is coherent if it comes from the same luminous point. It should be noted that coherence is not a statement about the luminous object, but about the diffracted image. A simple experiment to check this definition can easily be devised. If a small aperture is illuminated with light from two remote light sources, that are close enough together so that the aperture does not resolve them as two luminous points, interference can then be produced in the light passing through the aperture.

It is interesting that the introduction of wave length alone already makes comprehensible some phenomena which are usually explained by Heisenberg's indeterminacy principle. The canonical variables in optics are the coordinates of the position of a luminous point and the coordinates of the normal vector $\vec{n}$, which in an isotropic medium has the direction of the ray.

Because of diffraction, the position of the light source cannot be determined exactly, since each object point is seen as a little disk. Referring to the fact that the diameter of the disk is $1.22\lambda/A$, we see that the accuracy with which the object point can be located increases with increasing aperture. However, the light falling on the image plane comes from all points of the aperture. If we intend to select a

special direction, we must reduce the size of the aperture, which, in turn, increases the size of the Airy disk, and therefore the inaccuracy of determining the position of the luminous point.

Thus, "accuracy" in the position of the luminous point and "accuracy" in the direction of a light ray cannot be attained simultaneously. However, in diffraction theory there is a difference between the reality of the position of a light source and the direction of a "light ray."

At this time we may mention the part played by probability in quantum theory, applying it to our particular optical problem. In the special diffraction problem studied here, we see instead of a point a little disk, the light distribution of which is given by the Bessel function $J_1(z)$. It is philosophically possible to speak of the probability of the position of the object point, but I feel that such a description does not quite fit the facts.

First, since only the image is open to investigation, we really know nothing about the object except that it is located somewhere within the Airy disk projected into the object space. There is certainly no object point in the mathematical sense, but one or more extended objects, and therefore no exact point. The position of the object can be determined with a certain finite accuracy: it will be located well within the projected Airy disk, and never outside. Thus, its probable position is certainly not given by a Gaussian distribution, and not even by the Bessel function which describes the light distribution in the image. The Bessel function has finite values outside the zero values, but nobody would locate the object in the projected rings around the Airy disk. Also, the finite wave length results in an *inaccuracy*, and not an *indeterminacy*.

At this stage, I should like to digress a moment to call attention to the fact that another use of the Gaussian distribution is philosophically not quite justified. Many physical experiments lead finally to the measurement of a certain quantity. To make this measurement, a scale is used. Measuring the quantity many times, we will get a series of results dispersed around a certain probable value. Such a distribution is frequently assumed to be Gaussian. As long as only an approximate value is wanted, this might be perfectly satisfactory. Still, we should not lose sight of the fact that, in a philosophical sense, the Gaussian distribution does not give the correct picture, since the real value is always ascertained with a *finite* inaccuracy. For instance, the length of my finger will not measure a mile, even if it is measured a billion times. I feel that this is one point where the model leads to false philosophical conclusions. The physicist does not really believe in the Gaussian distribution either, since he always discards from his calculations any data showing "accidental errors," i.e., highly improbable readings.

We have discussed the simplest optical problem in considerable de-

tail, because it seems to have all the important qualitative features. Of course, aberrations of the optical system can easily be superimposed. Quite a number of problems in quantum mechanics can also be treated on this level, since the discovery of diffraction for mass particles has been verified by Davisson and Germer, whose experiments allow a wave length to be coordinated with a mass particle.

The following picture, derived from an idea of Euclid, may possibly be of value in visualizing the physical facts on the second level. There is obviously, a certain reciprocity between the light source and the aperture of an optical system. Since the aperture defines a certain area at the light source as equivalent to a point, the light source considered as an equally luminous area defines a certain "point" in the aperture plane. The connection of two such extended "points" can be called a "light ray." In this sense, there are always only a *finite* number of light rays. The manner in which this definition can be used to understand the different problems and ideas in the image formation of a microscope has been discussed in one of my previous papers.[5]

The problem of diffraction at a circular aperture has been discussed. On this level we might also investigate diffraction at a plane grating and finally diffraction at a spatial grating.

I suspect that the theory of diffraction at a spatial grating will explain the dependence of refractive index on wave length; in other words, the dispersion of optical materials. Since the mathematical development of such a dispersion theory seems quite difficult, I suggest a series of experiments to test this idea. The development of radar has given us sources of electromagnetic radiation with wave lengths of the order of a few centimeters. Models could be made of the atomic structure of optical materials, especially crystals, on a proportional scale, and the phenomena of diffraction and refraction could be studied at this great magnification.

It is not impossible that a formula for refractive index as a function of wave length might be found with such a model. In support of this assumption seem to be the theory of thin layers and the results of my empirical investigation of dispersion formulas. To layers that are only a few wave lengths thick, a refractive index can be ascribed that lies between unity and the index of the same material in finite layers. The empirical investigation of dispersion curves showed that the refractive index μ is much better approximated than μ^2 by an equation of the Sellmeier type, namely,

$$\mu = \Sigma \frac{a_\nu}{\lambda^2 - \lambda_\nu^2}. \qquad (21)$$

[5] M. Herzberger, "Theory of Microscope Vision," J. O. S. A., 26, 52 (1936).

It is my belief that the refractive index itself has a definite physical meaning, and that the absorption bands indicated in equation (21) are associated with the structural constants of the material.

THE THIRD LEVEL.

The first level can be described mathematically as pure geometry; the second level, as a boundary value problem; and the third level like the second but in a different space. In the second level we introduced the wave length λ, and this was sufficient for the investigation of all stationary problems. In the third level we must take into account the fact that the transport of energy takes time, and that its velocity c is an absolute physical constant. In describing optical phenomena, the quantity $ict = x_4$ is introduced as a new variable, and $ics = x_{n+1}$ for mechanical problems. The various phenomena are then described in the new space by the same equations that apply to the second level.

I have philosophical objections to using in physical problems, which are real, the imaginary quantity $i = \sqrt{-1}$. I prefer to describe the problems in a space having real coordinates, but a hyperbolic measure:

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2. \tag{22}$$

This is quite feasible. Another possibility is to set Minkowski space in a five-dimensional continuum, in which we study only a four-dimensional hyperbolic subspace.

However, for the purpose of this preliminary exposition, it might be easier to adopt Minkowski's treatment, since the literature in which it is used is extensive, and the results can be used in bringing out the relevant points.

In Minkowski space, the normal vector is a contragradient vector with $\binom{n+1}{2}$ components, where n is the number of degrees of freedom. For the original optical problem, or for a mass point, the vector has six components. This is equivalent to Dirac's "spin" theory of an electron, and leads directly to the electromagnetic theory of light, as described, for instance, by H. Minkowski and M. Born.[6]

By again introducing wave length, we can solve diffraction problems, which now become, like all physical laws, invariant for a generalized Lorentz transformation in optical space.

This treatment leads us to suspect the presence of "spatial gratings" in this space, corresponding to the structure of the static space previously considered. It is obvious that the introduction of wave length in this space results in the well-known relation between wave length and frequency:

$$\lambda\nu = c. \tag{23}$$

[6] H. Minkowski, "Zwei Abhandlungen über die Grundgleichung der Elektrodynamik," B. G. Teubner, 1910. See appendix written by M. Born.

We also obtain the other consequences of relativity, particularly the connection between mass and latent energy.

Instantaneous transport of energy is assumed if we set $ct \to \infty$, or $1/ct = 0$. In this case, all the laws of the third level are transformed into laws of the second level.

The theory up to this point is strictly causal, although not in the usual sense. It is causal in the relativistic optical space, where the energy transport designates consecutive stages.

The theory is a continuum theory, insofar as it permits passing from object to image. By the use of diffraction theory, it is possible to replace the unknown and unknowable object by a model object which can be used as the starting point of a calculation, the results of which will agree at each point with experiment.

THE FOURTH LEVEL.

On the fourth level, we add the simple experimental fact that energy absorption is always observed in steps, and that these steps are integral multiples of $h\nu = hc/\lambda$, where h is Planck's constant. We see that these steps are small for long wave lengths, and more pronounced for shorter wave lengths.

It is highly significant that there is on this level the threshold constant h, which is completely independent of wave length and is the same for all matter. This sets a finite limit to our ability to investigate radiation of very short wave lengths. However, we should not forget that we experience only the final absorbed image, and that the idea of "energy quanta" freely moving in space is a fictional model we make of an unknowable object.

Since this idea undoubtedly leads to contradictory results, as obtained for instance in Schrödinger's [7] well-known experiment, it might be wise to reconsider the experimental evidence and to attempt to revise the theory, carefully avoiding statements about the unknowable object.

It is my feeling that the Einstein effect alone, with the knowledge gained on lower levels, might suffice for the derivation of most of the experimental results of quantum mechanics, and I intend to review the experimental material with this idea in mind, at a later time.

CONCLUSION.

Although the ideas presented here are the results of thoughtful analysis rather than of any new experimental evidence, I felt that it might be worth while to present them for the critical judgment of theoretical physicists.

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[7] E. Schrödinger, "Über die Kohärenz in weitgeöffneten Bundeln," *Ann. d. Physik.*, 61, 69 (1920).

Any scientist should analyze his problems so as to separate gross effects from the effects due to fine correction, and should try to devise experiments in which the effects searched for are the gross effects. In this way, physical theory can remain relatively simple. The physicist must always be aware of the fact that, whatever he does, any measurement has only finite accuracy, that he can investigate an object only as seen through an instrument, and that the instrument has a finite aperture.

The fact that energy transport has a finite velocity independent of the observer makes it possible to order physical events independently of the observer. Thus, causal succession becomes a philosophical possibility in relativistic space.

The well-known connection of the value of $div\,\vec{n}$ with the existence of discontinuities in the field has not been discussed here. The theory is still in principle a dual theory, distinguishing between matter and energy. This comes from the fact that energy transport takes place with the velocity of light, while material particles do not have such a velocity. There is the possibility of gradual transition, but there should be left open the possibility that what we call matter may be composed of an ultimate particle plus a surrounding electrical field, and that physical phenomena are interactions between fields alone.

There are not many intrinsic constants in the theory. Refractive index and mass should both be derived constants; the first is related to the granular structure of matter, the other to the energy flux through a closed surface element.

In closing, I wish to reiterate that the purpose of this analysis has not been to find a complete solution for all physical problems, but to suggest a method for analyzing them.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

WARTIME RADIO OBSERVING PROJECTS.

During the war, radio amateurs of the United States participated in three observing projects sponsored by the Bureau, the object of which was to obtain basic data urgently needed in making radio propagation predictions for the Allied Armed Forces. The first two projects called NBS-ARRL projects I and II, were organized under the joint auspices of the National Bureau of Standards and the American Radio Relay League, the national organization of the radio amateurs of the United States, and most of the participating observers were ARRL members. The third project, known as the WWV Observing Project, was organized independently of the NBS-ARRL Projects, but was later joined by a number of NBS-ARRL Project II participants.

NBS-ARRL Project I was started on July 1, 1941, and terminated on June 30, 1943. Project II ran from July 1, 1943, to June 30, 1944. "Regional Coordinators," selected by the ARRL headquarters staff, organized amateurs in their vicinities into observation groups for monitoring the radio-frequency spectrum between 1,500 and 30,000 kilocycles for two 2-hour periods each week, one in the daytime around noon, and one after dark.

The intensity, readability, and background noise intensity, as well as the distance and direction of each observed station, were reported to the Bureau, where the data were analyzed and the results used in checking predictions of world-wide radio propagation conditions for the Allied Armed Forces and commerical radio communications companies engaged in war work.

The WWV Observing Project was organized in December 1943 and terminated on June 30, 1945. Its object was to obtain data on the reception of the Bureau's standard-frequency broadcast station WWV at various distances and in different directions. Observers were recruited from amateur and professional radiomen who had expressed an interest in the WWV broadcasts, and later from participants in the NBS-ARRL Projects. Observers were requested to tune in on WWV as often as possible at approximately one-hour intervals and to record ratings of the intensity, readability, fading, and background noise intensity. The results of this project were also used in checking predictions of world-wide radio propagation conditions.

Most of the participants in these projects had regular full-time jobs and were therefore able to make their observations only during spare

* Communicated by the Director.

time. Many went without sleep to maintain their observing schedules and used precious hours on Sundays and days off, but they did it gladly as a patriotic service without thought of pay or recognition.

Through these projects United States radio amateurs contributed data to the war effort that could have been obtained in no other way. In addition, they have increased the sum total of knowledge of radio wave propagation and communication, and this will benefit everyone in days to come.

HIGH POLYMERS—A NEW FIELD OF SCIENCE.

Fundamental research on seemingly diverse products—rubber, textiles, paper, leather, and plastics—is being integrated through the application of the methods and techniques of the new science of high polymers. Basically, the different substances made up of large organic molecules have much in common whatever their practical form and application. The same apparatus and methods can ordinarily be used to measure the various electrical, thermal, thermodynamic, and optical properties. Likewise, many of the same procedures and the same reactions are used in the determination of the composition, molecular weight, molecular dimensions, and structure of all the large molecules.

The molecular properties of high polymers, far from being only of academic interest, are rapidly becoming of commercial importance. Various synthetic plastics and rubbers are now being bought and sold on the basis of molecular weight and related properties. Hence, it is necessary for the Bureau to acquire equipment and develop techniques for making these measurements, and to establish standards for the use of science and industry.

In order to keep pace with developments in this new field, a Colloquium on High Polymers is being conducted as a part of the Bureau's educational program. Meetings are held one evening each week under the sponsorship of Dr. Robert Simha, a well-known authority on the subject who has recently joined the staff of the Organic and Fibrous Materials Division.

The speakers at the Colloquium include scientists from outside Washington who are working in academic institutions or industrial research laboratories, and also members of the staff of the Bureau, the Naval Research Laboratory, the Office of Rubber Reserve, the Milton Harris Associates, and the local universities. The topics covered have been coordinated under the general headings—(1) properties of solutions of high polymers, (2) physical properties of rubber, plastics, and fibers, and (3) the mechanism of formation and degradation of long chain compounds. These topics have been selected to cover fields in which the Bureau is actively engaged or is planning future work.

Out-of-town speakers at the Colloquium include some of the foremost workers in the field, and when they are in Washington arrangements are usually made for them to spend additional time in consultation on the Bureau's work in their respective fields of specialization. The expenses of these speakers are defrayed by a fee paid by the regular members of the Colloquium, and administered by the Bureau Educational Committee. Occasional visitors are welcomed to the Colloquium without charge.

CHEMICAL CONSTITUTION OF PORTLAND CEMENT CLINKER.

The chemical constitution of portland cement clinker in terms of definite compounds has been one of the principal subjects of study by the Portland Cement Association Fellowship at the Bureau. Recently, the work has centered in investigations of the manner in which the alkalies, Na_2O, and K_2O, are combined in clinker. One phase of this research is reported in a paper by K. T. Greene and R. H. Bogue (RP1699) that will be published in the February number of the *Journal of Research*.

The region of the system that bears upon commerical cement compositions was examined and the various stable crystalline phases were determined. Particular attention was given to the compounds of Na_2O, or phases containing Na_2O, that are formed in the part of the above system that approaches portland cement clinker in composition. The results of the phase investigation are presented in tables and are shown graphically in phase diagrams.

It was found that the Na_2O may occur in various ways depending on the composition of the mixture and the heat-treatment it receives. Under certain conditions it may be combined as the compound $Na_2O.8CaO{\cdot}3Al_2O_3$, or as a solid solution of this compound with $3CaO{\cdot}Al_2O_3$, which is a constituent of clinker. If cooling is rapid so that glass is produced, some of the Na_2O will be found in this glass. At high temperatures $2CaO{\cdot}SiO_2$, a constituent of clinker, can take up Na_2O and other oxides in solid solution, later precipitating them as inclusions of a Na_2O-bearing phase when the $2CaO{\cdot}SiO_2$ undergoes inversion from the α to the β crystalline form.

The results of the work indicate the manner in which Na_2O may be combined in portland cement clinker, but the conclusions are limited by the exclusion of certain components present in clinker, notably Fe_2O_3.

PURIFICATION AND PROPERTIES OF 51 HYDROCARBONS.

The February number of the *Journal of Research* contains a report (RP1695) on the purification and properties of some paraffin, alkylcyclopentane, alkylcyclohexane, and alkylbenzene hydrocarbons, by

Alphonse F. Forziati, Augustus R. Glasgow, Jr., Charles B. Willingham, and Frederick D. Rossini. The investigation forms part of the American Petroleum Institute's Research Project 6 on the "Analysis, purification, and properties of hydrocarbons." The report describes the purification and gives results of measurements of refractive index (n_D at 20° and 25° C.), density (at 20° and 25° C.), boiling point and pressure coefficient of the boiling point (at 1 atmosphere); and (except for 4 of the compounds) the freezing point, together with the calculated amount of impurity, of samples of 51 hydrocarbons, including 29 paraffins, 4 alkylcyclopentanes, 10 alkylcyclohexanes, and 8 alkylbenzenes.

PRODUCER GAS AS A SUBSTITUTE FOR MOTOR FUEL.

A series of tests has been made at the Bureau by Clarence S. Bruce and F. A. Middleton on a four-cylinder stationary engine equipped to operate on either gasoline or producer gas from charcoal. These tests were made to determine the comparative consumption of charcoal and gasoline under various loads at engine speeds from 900 to 1,800 revolutions per minute. The optimum spark advance was determined for each fuel and operating condition. To illustrate the relative performance with gasoline and charcoal more graphically, the test data were used to calculate the road performance of a light truck which might logically be powered by the test engine. The "truck" used in these calculations was a 1½ton model with 6.00-16 tires and a 4.16 rear axle ratio.

As set forth in the report on this work (RP1698), which will be published in the February *Journal of Research*, the calculations showed a gasoline mileage of 31.7 miles per gallon with an empty truck at 18 miles per hour, and 23.2 miles per gallon at 36 miles per hour. With the same truck fully loaded, the gasoline mileages at these speeds were respectively 24.2 and 19.3. On charcoal it was calculated that the same empty truck, would travel 2.8 miles at 36 miles per hour, and 2.0 miles at 18 miles per hour for each pound of charcoal burned. At the same speeds the distances travelled with the loaded truck per pound of charcoal burned would be respectively 2.1 and 1.6 miles. It was found that one gallon of gasoline was approximately equal to 11 pounds of charcoal in operating a gasoline engine of a common type. The performance of an engine using charcoal producer gas could be improved by supercharging.

To obtain maximum power with producer gas, the spark had to be much more advanced than with gasoline. The maximum power of the engine was reduced approximately 45 per cent. There was considerable trouble in operation because of clogging of filters in the gas line. Starting usually required about 2 minutes, and with proper technique was not a problem.

Servicing requirements would make the use of gas producers impractical for the average automobile owner in this country. The loss in maximum engine power would place limits on loading that might be a serious handicap, but would not prevent the use of producer gas as a substitute fuel. However, the authors of the paper feel that their work indicated that operation on any liquid fuel would be preferable to operation on charcoal.

DETERMINING SMALL AMOUNTS OF GOLD.

Many articles, especially jewelry, have on the surface a thin electrodeposited layer of gold. This layer, which may vary in thickness from less than one-millionth to more than a ten-thousandth of an inch, not only produces an attractive color but also retards corrosion and tarnish of the base metal.

Since the wearing qualities of these articles largely depend on the thickness of the gold layer, an accurate method of determining it is desirable. In the *Journal of Research* for February (RP1694), W. Stanley Caabaugh describes such a method. A sample of the object to be tested is obtained by means of a punch and die designed to cut out samples of small area (1 square millimeter or 0.00155 square inch) from material that has a total thickness of 1/32 inch or less. The sample cut out with the punch is treated with dilute nitric acid, which dissolves the base metal and leaves a small circular piece of gold. This is washed by decantation several times with water, and is then dissolved in a few drops of aqua regia, after which it is evaporated to dryness by impinging on the surface of the solution a jet of purified air. The quantity of gold from which the thickness of the gold layer is calculated is determined with a spectrophotometer from the intensity of yellow color produced when a measured volume of a solution of *o*-tolidine (3-3′-dimethylbenzidine), dissolved in normal sulfuric acid, is added to the air-dried residue.

Quantities of gold up to 10 micrograms (0.010 milligram), corresponding to 0.00050 millimeter (0.00002 inch) of gold on 1 square millimeter (0.00155 square inch) of surface, can be determined accurately by this method.

Nine Pounds of Penicillin.—Describing the whole penicillin development as a monumental undertaking, Robert D. Coghill of the U. S. Department of Agriculture recently gave figures on the cost of the commercial development and the weight of the output.

There were, in the spring of 1945, about 13 penicillin plants in the United States and Canada with a total investment of about 20 million dollars. Based on a tentative production program of 200 billion units a month, and on the potency assigned to the National Standard, these $20,000,000 worth of plants will produce only 9 pounds of pure penicillin a day. Yet this quantity will treat approximately 250,000 serious cases a month. For the fighting men it has meant the saving of thousands of lives—and of arms and legs.

Discussing the cost of penicillin treatment, Dr. Coghill said the first quoted price was $20 for 100,000 units which by April 1944 had declined to $3.25 for 100,000 units and in August 1945 the quotation was 59 cents. On the average, it takes about 1,000,000 units to treat a serious case of one of the infections for which penicillin treatment is effective.

Dr. Coghill is head of the Fermentation Division of the Bureau of Agricultural and Industrial Chemistry's Northern Regional Research Laboratory, Peoria, Ill., which contributed materially to the development of the methods that made commercial production possible. It was at this laboratory that research workers found the way to feed the molds with corn steeping liquor and milk sugar, which coupled with the use of better strains of molds, stepped up the production of penicillin more than 100-fold. The Laboratory workers also adapted to penicillin production the more efficient submerged culture of the mold along lines developed earlier with other commercial fermentation processes. They also tested and found other species and strains of the mold which are more productive. Dr. Coghill says that the mold most productive in surface fermentations is not a good producer in submerged cultures, and the best submerged culture mold is not efficient for surface cultures.

R. H. O.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, WEDNESDAY, FEBRUARY 20, 1946

The Franklin Institute held its Stated Monthly Meeting on Wednesday, February 20, at 8:15 P.M. in the Hall of the Institute, Mr. Charles S. Redding, President, presiding.

The meeting was called to order after the playing of the National Anthem.

The members were asked to approve the minutes of the December meeting which appeared in full in the January JOURNAL. Upon motion the Minutes were approved as printed.

The Secretary, Dr. Henry Butler Allen, reported on Membership as follows:

Active	62
Associate	41
Student	33
Total	136

He then reported that the Board of Managers at its meeting in the afternoon had nominated for Honorary Membership in The Franklin Institute, upon recommendation of the Committee on Science and the Arts, the two Franklin Medalists for 1946, namely:

Dr. Henry Clapp Sherman	Sir Henry Thomas Tizard, *President*
Columbia University	Magdalen College
New York City	Oxford, England

The Secretary having put his statement in the form of a motion, which was duly seconded, the President put it to vote. The two gentlemen were unanimously elected to Honorary Membership.

The speaker of the evening, Colonel Julian Z. Millar, Radio Research Engineer, The Western Union Telegraph Company, New York, was introduced by the President, who gave a brief summary of Colonel Millar's activities.

Colonel Millar then gave his talk entitled "A Preview of the Western Union System of Radio Beam Telegraphy," discussing in detail Western Union's plans to establish super-high-frequency radio relay telegraph systems between the major cities in the United States during the next seven years. Colonel Millar stated the plan began in 1945 and illustrated with slides what could be expected by 1952.

After the Lecture he displayed some apparatus which will be in use in this new project.

The meeting was dismissed with a rising vote of thanks at 9:50 P.M.

HENRY BUTLER ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, February 13, 1946.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 13, 1946.

DR. HIRAM S. LUKENS, *in the Chair.*

The following reports were presented for final action:

No. 3157: Work of G. M. Barnes.

This report recommended the award of an Elliott Cresson Medal to G. M. BARNES, Ordnance Department, Washington, D. C., "In consideration of his notable contributions to the design and development of numerous ordnance items, particularly anti-aircraft guns, tanks, and seacoast artillery; for his early development of welded gun carriages; and for his masterly organization of ordnance research, closely articulated with development and production."

No. 3161 } The Standing Sub-Committee on the Franklin Medal recommended
No. 3162 } that a Franklin Medal be awarded in 1946 to:

HENRY CLAPP SHERMAN, of Columbia University, New York, "In consideration of his many contributions to the science of nutrition and especially of his many public services, as a result of which the general health level of millions of human beings all over the world has been uplifted;" and one to

HENRY THOMAS TIZARD, of Oxford, England, "In recognition of his outstanding contributions, which have advanced the science of aeronautics in the fields of full scale flight test procedure; internal combustion engines relating to fuel economy and for his leadership in aeronautical research and development which, in part, involved the development of special devices for air offensive operations and for defense against an attack."

No. 3165: Levy Medal.

This report recommended the award of the Louis Edward Levy Medal to GEORGE CLARK SOUTHWORTH, of Red Bank, New Jersey, "For his paper 'Microwave Radiation from the Sun,' which appeared in the April 1945 issue of the Journal."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays from two until ten o'clock P.M.

RECENT ADDITIONS.

ALMANACS.

Bulletin Almanac. 1946.

ARCHITECTURE AND BUILDING.

GEESON, ALFRED G. Building Science. Volume I. 1944.
SITTE, CAMILLO. The Art of Building Cities. 1945.

BIOCHEMISTRY.

HÖBER, RUDOLF, AND OTHERS. Physical Chemistry of Cells and Tissues. 1945.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

British Chemical Abstracts. Collective Index A and B. 1933–1937. Parts 1-2. No date.
LEFFINGWELL, GEORGIA, AND MILTON LESSER. Glycerin. 1945.
WARD, A. G. Colloids. 1945.
WELLS, A. F. Structural Inorganic Chemistry. 1945.
YOUNG, C. B. F., AND K. W. COONS. Surface Active Agents. 1945.

ELECTRIC ENGINEERING.

APPLEYARD, ROLLO. The History of the Institution of Electrical Engineers (1871–1931). 1939.
FREEDMAN, SAMUEL. Two-Way Radio. 1946.
HILLS, STANLEY M. Battery-Electric Vehicles. 1943.
STOKLEY, JAMES. Electrons in Action. 1946.
TURNER, RUFUS P. Radio Test Instruments. 1945.
ZWORYKIN, V. K., AND OTHERS. Electron Optics and the Electron Microscope. 1945.

GRAPHIC ARTS.

KEPES, GYORGY. Language of Vision. 1944.
Photograms of the Year 1946.
TUPHOLME, C. H. S. Photography in Engineering. 1945.

MATHEMATICS.

BLASCHKE, WILHELM. Vorlesungen über differential Geometrie I. 1945.
COOLIDGE, JULIAN LOWELL. A History of Geometrical Methods. 1940.

MECHANICAL ENGINEERING.

ABBOTT, W. Machine Drawing and Design. Fourth Edition. 1945.
CHAMBERS, B. H. Twist Drills. 1944.

PHYSICS.

Institute of Physics. Industrial Radiology Group. Handbook of Industrial Radiology. Edited by J. A. Growther. 1944.

SCIENCE.

Royal Society of London. Philosophical Transactions. Series A, Volume 238. 1940.
U. S. Office of Scientific Research and Development. Science the Endless Frontier: a Report to the President by Vannevar Bush. 1945.

STATISTICS.

BLANKENSHIP, ALBERT B. Consumer and Opinion Research. 1943.

SUGAR.

Sugar Reference Book and Directory. Volume 14. 1945.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Abstract of **Ribonucleinase. I. Manometric Determination of Ribonucleinase in Blood and Tissues of the Rat and the Rabbit.**—CHARLES A. ZITTLE AND ELIZABETH H. READING (*Journal of Biological Chemistry*, **160**: 519, 1945). The ribonucleinase contents of the blood, plasma, and blood cells of the rat and the rabbit were determined by the method of Bain and Rusch (*Journal of Biological Chemistry*, **153**: 659, 1944). Most of the enzyme was found to be in the blood cells. Assays were also performed on the bone marrow, spleen, and pancreas of the rat and the rabbit. Solubility studies yielded confirming evidence that the nucleic acid is broken down by the blood.

Abstract of **Ribonucleinase. II. Mononucleotides in Commercial Ribonucleic Acids and Their Effect on Ribonucleinase.**—CHARLES A. ZITTLE (*Journal of Biological Chemistry*, **160**: 527, 1945). The difference in reactivity between the two types of commercial nucleic acid preparations (free acid and sodium salt) with ribonucleinase has been correlated with the difference in mono- and tetranucleotide content. The increased reactivity of the free acid with ribonucleinase after precipitation with acetic acid has been ascribed to the removal of these substances, since ribonucleinase was inhibited by a mixture of mononucleotides, by adenylic and by guanylic acids, and since the tetranucleotides are expected to be less reactive than a polynucleotide. The inhibition of ribonucleinase by mononucleotides was probably due to the competition of the mononucleotides with the substrate for the enzyme. This inhibition was about the same with several different concentrations of nucleic acid.

BOOK REVIEWS.

A Collection of Papers in Memory of Sir William Rowan Hamilton. The Scripta Mathematica Studies, Number Two. 82 pages, 17 × 25 cms. New York, Scripta Mathematica, 1945. Price $1.50.

The year 1943 marked the one hundredth anniversary of the discovery of quaternions by Sir William Rown Hamilton. Unfortunately the war prohibited fitting celebration at the time, so this volume has been prepared in his honor.

J. L. Synge discusses the life and early work of Hamilton emphasing his other great idea—the principal or characteristic function. Algebra's debt to Hamilton forms the subject of C. C. MacDuffee's paper. He points out that the presentation of the complex number field as a theory of couples of real numbers was a more fundamental concept in algebra than the spectacular quaternion. An elementary explanation of the theory of quaternions is offered by F. D. Murnaghan.

In the field of dynamics Hamilton also made important contributions. A summary of his work in this field and its subsequent influence on modern thought is presented by H. Bateman. The concluding article by Vladimir Karapetoff is entitled "The Constancy of the Velocity of Light."

The various papers here gathered form an appropriate tribute to this great Irish mathematician.

George E. Pettengill.

The Chemical Constituents of Petroleum, by A. N. Sachanen. 451 pages, tables and illustrations, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $8.50.

The subject of this book is one in which considerable work has been done but where a broad unexplained field exists. Many problems, particularly in the field of chemistry of lubricating oils, petroleum wax, resins and asphaltic constituents have not been crystalized into commonly accepted theories. New methods and conspicuous improvements in classical methods open new possibilities in attacking problems related to the composition of petroleum. The enormous complexity of petroleum, though rather a drawback to investigation today may become a great stimulus tomorrow.

The work opens with a treatment of the chemical composition of petroleum gases and the light gasolines produced from them and then proceeds into physical methods of determining hydrocarbons in distillates including distillation, crystallization, separation by solvents, adsorption and desorption, specific gravity and refractive index, refractory intersept etc. The chemical methods are next taken up and they include determination of the H:C ratio, ring analysis, sulfonation of aromatics, aromatic hydrocarbons, and fourteen others. In the coverage of hydrocarbons of straight run distillates it is pointed out that the commercial value of petroleum products is determined rather by their bulk chemical composition than by specific individual hydrocarbons and that "it seems certain however that the wide commercial segregation of individual hydrocarbons from liquid petroleum products is a problem of the near future."

Subsequent chapters are devoted to hydrocarbons of synthetic distillates, petroleum wax, oxygen compounds where reference is made that the origin of low molecular weight fatty acids still awaits solution, sulfur and nitrogen compounds knowledge of which is limited, resins and asphaltic compounds, and classification of crude oils. There is an author and subject index in the back.

The book is a review of a great amount of literature coupled with first hand knowledge of the subject, leading to the point where research can well begin.

R. H. Oppermann.

ADVANCING FRONTS IN CHEMISTRY, edited by Sumner B. Twiss. Volume I, High Polymers. 196 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $4.00.

In the Spring of 1944 as the first phase in the development of a High Polymer program at Wayne University, Detroit, there was presented a Symposium on High Polymers. This field is one of rapid development and there is need of correlation of recent and divers experimental information. The lectures are an attempt at such correlation, presenting unified concepts of certain phases of the polymerization problem as they are understood and interpreted by various experts in the field.

There are ten lectures in all, each having a different author. The first three are fundamental, dealing with molecular structure and the applications of catalysis to hydrocarbon reactions of importance in the synthesis of high polymers. The next two give a review of the direct and indirect evidence for the free radical mechanism of addition polymerization. Molecular size distribution in high polymers and the effect of chain length on physical properties of cellulose derivatives is discussed as is the study of the geometry of long chain molecules and the effect of inter-molecular forces as illuminated by X-ray diffraction studies. Recent work on the problem of orientation of chain polymers and their mobility in the liquid state is given through a study of the mechanical properties of their concentrated solutions. The last chapter of the book presents a discussion on some concepts of textile fibres—properties which make fibres useful and emphasize the dependence of some of these properties on the high polymer structures or molecular architectures of a number of the fibre systems.

There is a logical sequence to the arrangement of the lectures, and there is much food for thought and facts as a basis for further work presented here.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

You . . . and the Universe, by John J. O'Neill. 328 pages, 15 × 23 cms. New York, Ives Washburn, Inc., 1946. Price $3.50.

Radio Test Instruments, by Rufus P. Turner. 219 pages, drawings and illustrations, 16 × 24 cms. Chicago-New York, Ziff-Davis Publishing Co., 1945. Price $4.50.

Two-Way Radio, by Samuel Freedman. 506 pages, drawings and illustrations. 16 × 24 cms. Chicago-New York, Ziff-Davis Publishing Co., 1946. Price $5.00.

Encyclopedia of Chemical Reactions, by C. A. Jacobson. Volume 1. 804 pages, 15 × 24 cms. New York, Reinhold Publishing Corporation, 1946.

Petroleum Production, Volume 1, by Park J. Jones. 228 pages, drawings, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $4.50.

Electrons in Action, by James Stokley. 320 pages, drawings and illustrations, 14 × 21 cms. New York and London, Whittlesey House, McGraw Hill Book Company, Inc., 1946. Price $3.00.